LA FISICA DEI SISTEMI A MOLTI CORPI

Many-Body Physics

II SEMESTRE 9 CFU

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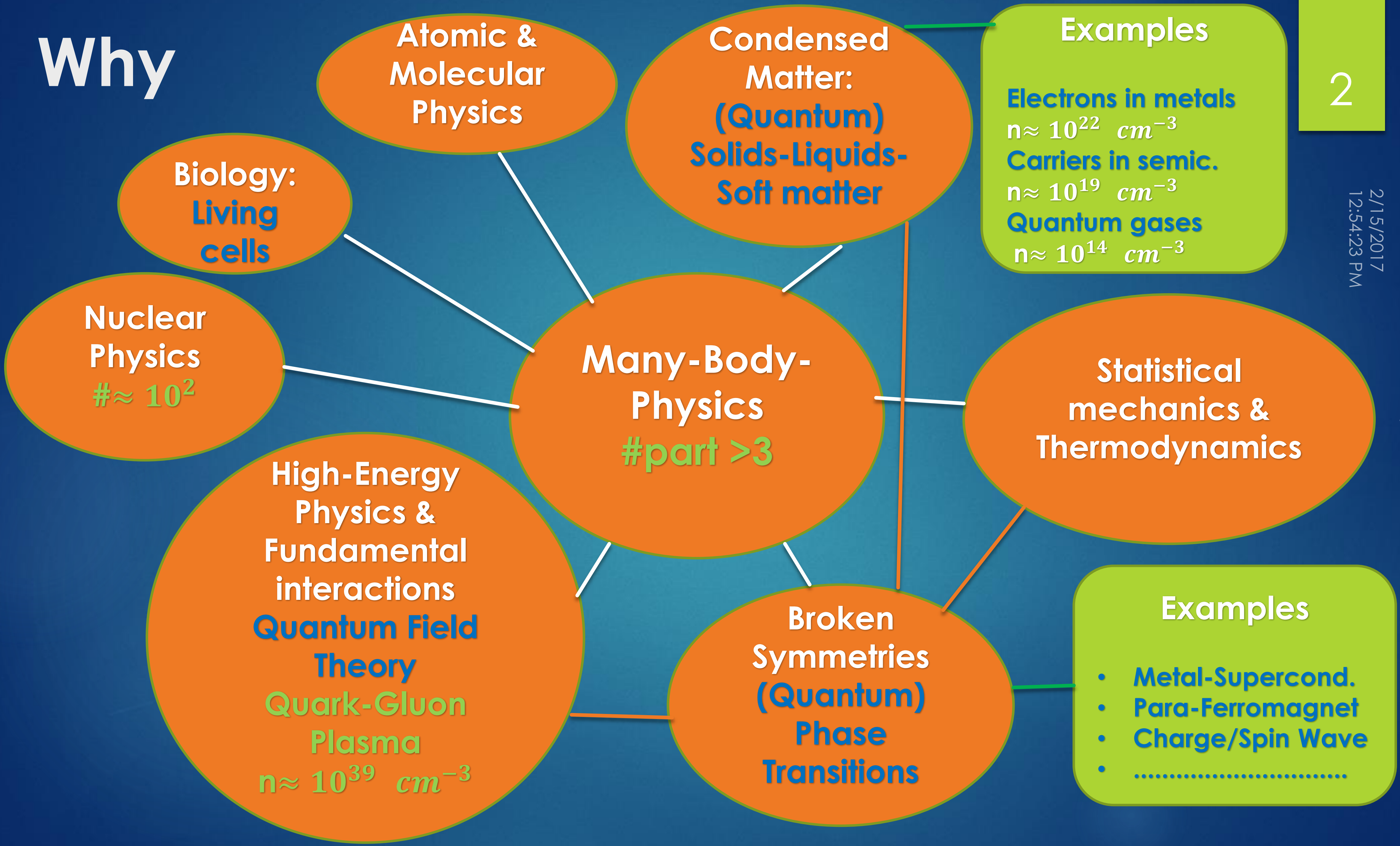
Why


Atomic & Molecular Physics
Condensed Matter: (Quantum) Solids-Liquids-Soft matter
Examples
Electrons in metals
n≈ 10^{22} cm^{-3}
Carriers in semic.
n≈ 10^{19} cm^{-3}
Quantum gases
n≈ 10^{14} cm^{-3}
Biology: Living cells
Nuclear Physics
#≈ 10^{2}
Many-Body-Physics
#part >3
Statistical mechanics & Thermodynamics
High-Energy Physics & Fundamental interactions
Quantum Field Theory
Quark-Gluon Plasma
n≈ 10^{39} cm^{-3}
Broken Symmetries (Quantum) Phase Transitions
Examples
Metal-Supercond.
Para-Ferromagnet
Charge/Spin Wave
....

What: a unifying idea via 2 CONCEPTS

CONSERVED QUANTITIES

- Number of particles
- Momentum/current (angular too)
- Energy
- ………..

+

BROKEN SYMMETRIES

- Liquid to Crystal
- Normal to Super Fluidity
- Para to Ferro Magnetism
- ………..

Classical → λ_{dB} ~ sistem size → Quantum

Appear:
- New Hydodynamic modes
- New elastic constants
- Defects

Reduced symmetry

Temperature

Interactions

Dimensionality

What: a unifying idea via 2 METHODS

QUANTUM FIELD THEORETICAL METHODS + **QUANTUM SIMULATIONS**

Classical → λ_{dB} ~ sistem size → Quantum

Appear:
- New Hydodynamic modes
- New elastic constants
- Defects

Reduced symmetry

Temperature

Interactions

Dimensionality

THE QUANTUM MANY-BODY PROBLEM: SETTING THE SCALES

$$\Psi(\vec{x}_1, \vec{x}_2, \ldots \vec{x}_N, t),$$

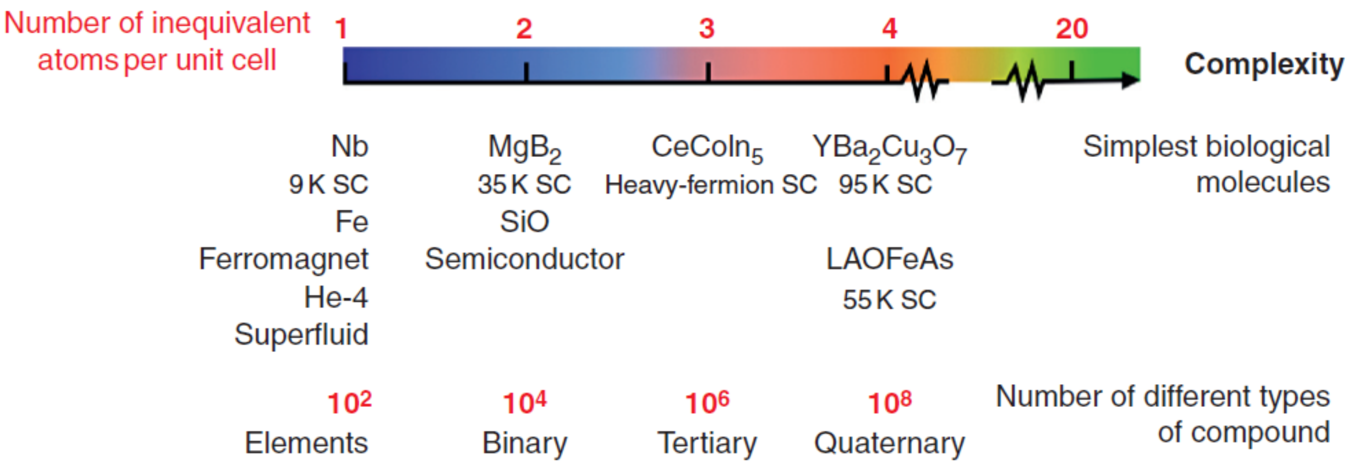
$$\left\{ -\frac{\hbar^2}{2m} \sum_{j=1}^{N} \nabla_j^2 + \sum_{i<j} V(\vec{x}_i - \vec{x}_j) + \sum_j U(\vec{x}_j) \right\} \Psi = i\hbar \frac{\partial \Psi}{\partial t}.$$

COMPLEXITY

1.4 C: complexity and emergence

Real materials are like “macroscopic atoms,” where the quantum interference among the constituent particles gives rise to a range of complexity and diversity that constitutes the largest gulf of all. We can attempt to quantify the “complexity” axis by considering the

Scales and complexity



Condensed matter of increasing complexity. As the number of inequivalent atoms per unit cell grows, the complexity of the material and the potential for new types of behavior grows. In the labels, “X K SC” denotes a superconductor with a transition temperature $T_c = X$; thus MgB_2 has a 35 K transition temperature.

TIME SCALE

1.1 *T*: Time scale

We can make an estimate of the characteristic quantum time scale by using the uncertainty principle, $\Delta\tau\,\Delta E \sim \hbar$, so that

$$\Delta\tau \sim \frac{\hbar}{[1\ \text{eV}]} \sim \frac{\hbar}{10^{-19}\ \text{J}} \sim 10^{-15}\text{s}. \qquad (1.4)$$

Although we know the physics on this time scale, in our macroscopic world the characteristic time scale $\sim$ 1s, so that

$$\frac{\Delta\tau_{\text{macroscopic}}}{\Delta\tau_{\text{quantum}}} \sim 10^{15}. \qquad (1.5)$$

To link quantum and macroscopic time scales, we must make a leap comparable with an extrapolation from the time scale of a heartbeat to the age of the universe (10 billion years $\sim 10^{17}$s).

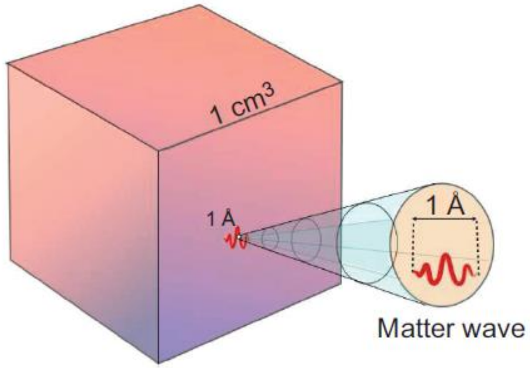
LENGTH SCALE

1.2 *L*: length scale

An approximate measure for the characteristic length scale in the quantum world is the de Broglie wavelength of an electron in a hydrogen atom,

$$L_{\text{quantum}} \sim 10^{-10}\text{m}, \tag{1.6}$$

1.4 *C*: complexity and emergence



The typical size of a de Broglie wave is 10^{-10} m, to be compared with a typical scale of 1 cm for a macroscopic crystal.

so

$$\frac{L_{\text{macroscopic}}}{L_{\text{quantum}}} \sim 10^{8} \tag{1.7}$$

(see Figure 1.1). At the beginning of the twentieth century, a leading philosopher-physicist of the era, Ernst Mach, argued to Boltzmann that the atomic hypothesis was metaphysical, for, he argued, one could simply not envisage any device with the resolution to detect or image something so small. Today, this incredible gulf of scale is routinely spanned in the lab with scanning tunneling microscopes, able to view atoms at sub-angstrom resolution.

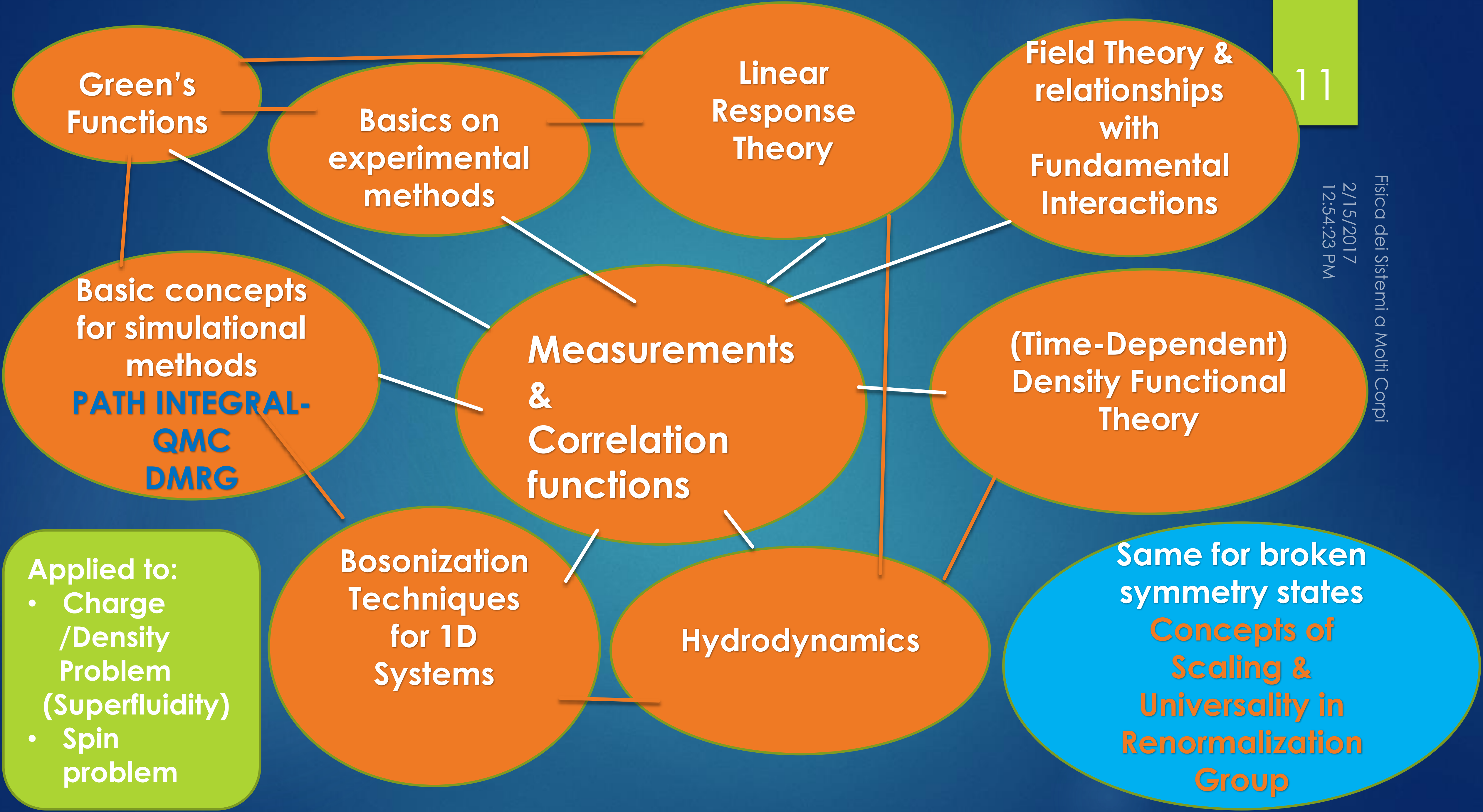
1.3 *N*: particle number

To visualize the number of particles in a single mole of a substance, it is worth reflecting that a crystal containing a mole of atoms occupies a volume of roughly 1 cm^3. From the quantum perspective, this is a cube with approximately 100 million atoms along each edge. Avagadro's number,

$$N_{\text{macroscopic}} = 6 \times 10^{23} \sim (100\ \text{million})^3, \tag{1.8}$$

is placed in perspective by reflecting that the number of atoms in a grain of sand is roughly comparable with the number of sand-grains in a 1-mile-long beach. But in quantum matter, the sand-grains, which are electrons, quantum mechanically interfere with one another, producing a state that is much more than the simple sum of its constituents.

What to Know & Do: Course Conceptual Map



Applied to:
- Charge /Density Problem (Superfluidity)
- Spin problem

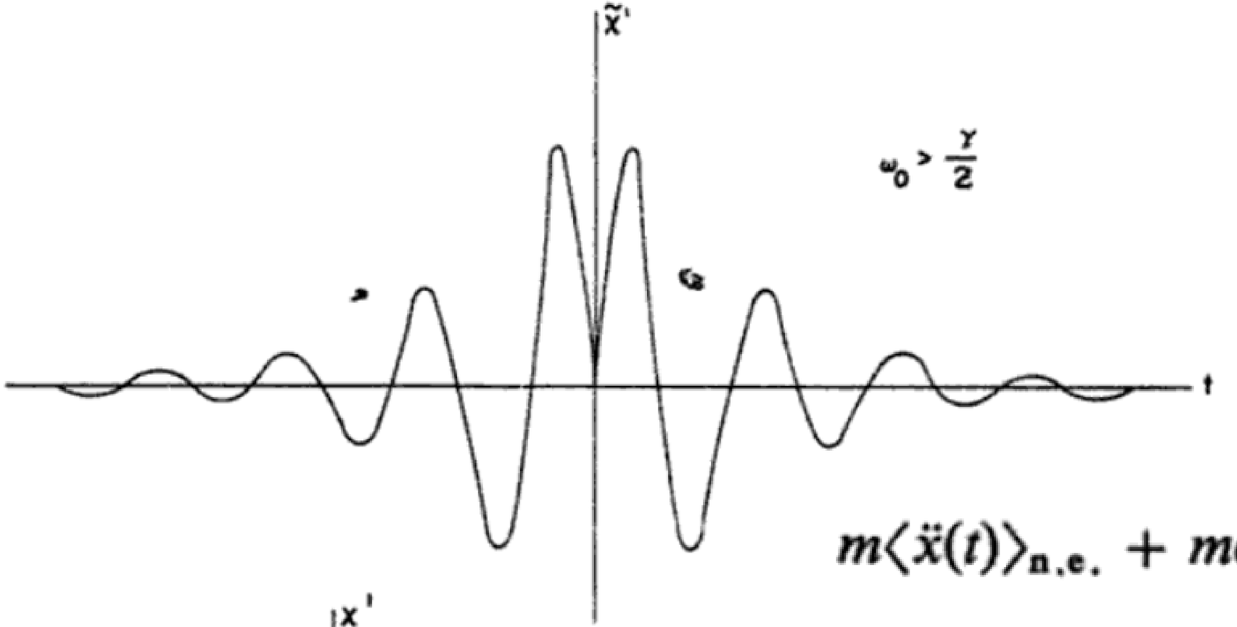
CONNECTING MACRO & MICRO

SELECTED COURSE EXAMPLES

MEASUREMENTS & EXP. METHODS

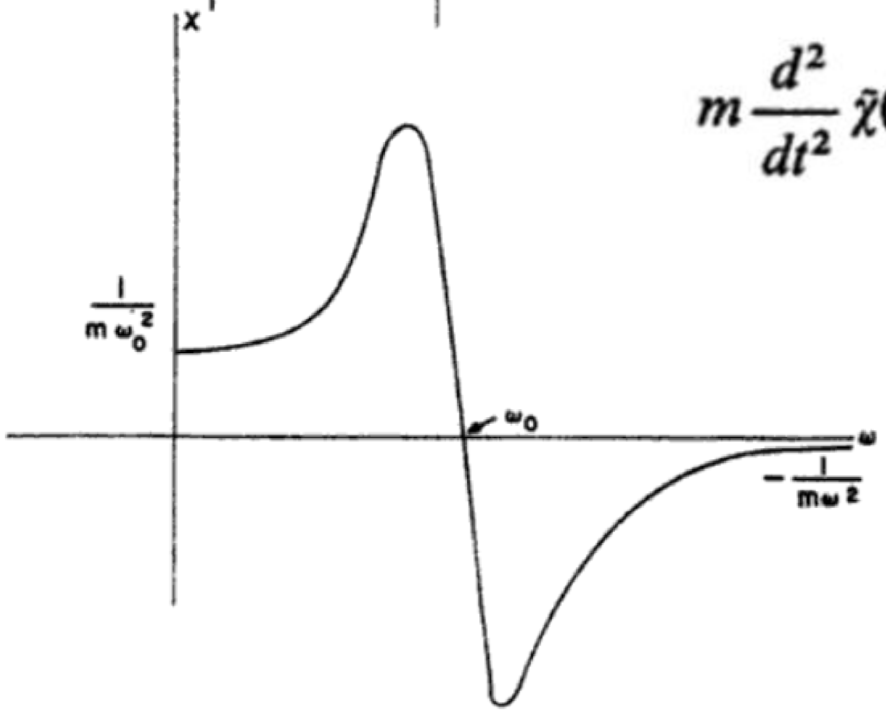
vs CORRELATION/RESPONSE/GREEN's FUNCTIONS
vs (QUANTUM) HYDRODYNAMICS
vs THERMODYNAMICS+STATISTICAL MECH.

MEASUREMENTS ETC…. I

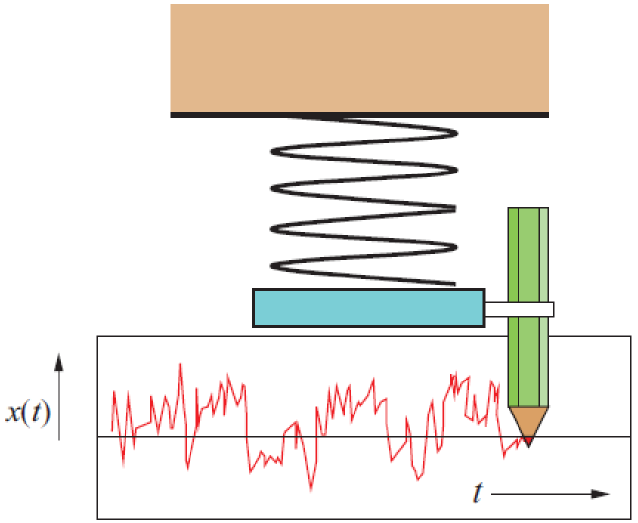


$$m\langle \ddot{x}(t)\rangle_{n.e.} + m\omega_0^2\langle x(t)\rangle_{n.e.} + m\gamma\langle \dot{x}(t)\rangle_{n.e.} = F^{ext}(t)$$

$$m\frac{d^2}{dt^2}\tilde{\chi}(tt') + m\omega_0^2\tilde{\chi}(tt') + m\gamma\frac{d}{dt}\tilde{\chi}(tt') = \delta(t-t')$$



Fluctuation-Dissipation Theorem



$$\overbrace{\langle x(t)x(0)\rangle}^{\text{fluctuations}} = 2k_B T \int \frac{d\omega}{2\pi} \underbrace{\frac{\chi''(\omega)}{\omega}}_{\text{dissipation}} e^{-i\omega t}$$

Table 9.1 Selected operators and corresponding response functions.

Quantity	Operator $\hat{A}^{a}$	$A(\mathbf{k})$	Response function
Density	$\hat{\rho}(x) = \psi^{\dagger}(x)\psi(x)$	$\rho_{\alpha\beta} = \delta_{\alpha\beta}$	Charge susceptibility
Spin density	$\vec{S}(x) = \psi^{\dagger}_{\alpha}(x)\left(\frac{\vec{\sigma}}{2}\right)_{\alpha\beta}\psi_{\beta}(x)$	$\vec{M}_{\alpha\beta} = \mu_B \vec{\sigma}_{\alpha\beta}$	Spin susceptibility
Current density[a]	$\frac{e}{m}\psi^{\dagger}(x)\left(-i\hbar\overleftrightarrow{\nabla} - e\vec{A}\right)\psi(x)$	$\vec{j} = e\vec{v}_{\mathbf{k}} = e\vec{\nabla}\epsilon_{\mathbf{k}}$	Conductivity
Thermal current[a]	$\frac{\hbar^2}{2m}\psi^{\dagger}(x)\overleftrightarrow{\nabla}\overleftrightarrow{\partial}_t\psi(x)$	$\vec{j}_T = i\omega_n\vec{v}_{\mathbf{k}} = i\omega_n\vec{\nabla}\epsilon_{\mathbf{k}}$	Thermal conductivity

MEASUREMENTS ETC… III

Table 9.2 Selected spectroscopies.

Type	Name	Spectrum	A	Notes and common measurement issues
ELECTRON	STM[a]	$\frac{dI}{dV}(\mathbf{x}) \propto A(\mathbf{x},\omega)\vert_{\omega=eV}$	$\psi(x)$	Surface probe. $T \sim 0$ measurement. Does the surface characterize the bulk?
	ARPES[b]	$I(\mathbf{k},\omega) \propto f(-\omega)A(\mathbf{k},-\omega)$	$c_{\mathbf{k}\sigma}(t)$	$p_\perp$ unresolved. Surface probe; no magnetic field possible.
	IPES[c]	$I(\omega) \propto \sum_{\mathbf{k}}[1-f(\omega)]A(\mathbf{k},\omega)$	$c^\dagger_{\mathbf{k}\sigma}(t)$	$\mathbf{p}$ unresolved. Surface probe.
SPIN	Magnetic susceptibility	$\chi_{DC} = \int \frac{d\omega}{\pi\omega}\chi''(\mathbf{q}=0,\omega)$	M	$\chi \sim \frac{1}{T+\theta}$, Curielaw: local moments. $\chi \sim$ constant paramagnet.
	Inelastic neutron scattering	$S(\mathbf{q},\omega) = \frac{1}{1-e^{-\beta\omega}}\chi''(\mathbf{q},\omega)$	$S(\mathbf{q},t)$	What is the background? Quality of crystal?
	NMR[d] Knight shift	$K_{contact} \propto \chi_{local}$	$S(\mathbf{x},t)$	How is the orbital part subtracted?
	Nuclear relaxation rate	$\frac{1}{T_1} = T\int_q F(\mathbf{q})\frac{\chi''(\mathbf{q},\omega)}{\omega}\Big\vert_{\omega=\omega_N}$		How does powdering affect sample?
CHARGE	Resistivity	$\rho = \frac{1}{\sigma(0)}$	$\vec{j}(\omega=0)$	How big is the resistance ratio $R(T=300\,\mathrm{K})/R(T=0\,\mathrm{K})$ of the sample?
	Optical conductivity	$\sigma(\omega) = \frac{1}{-i\omega}\left[\langle j(\omega')j(-\omega')\rangle\right]_0^\omega$	$\vec{j}(\omega)$	For optical reflectivity measurements: how was the Kramers–Kronig analysis done? Spectral weight transfer?

[a] Scanning tunneling spectroscopy.
[b] Angle resolved photoemission spectroscopy.
[c] Inverse photoemission spectroscopy.
[d] Nuclear magnetic resonance.

Thermodynamical derivatives
And
Transport coefficients

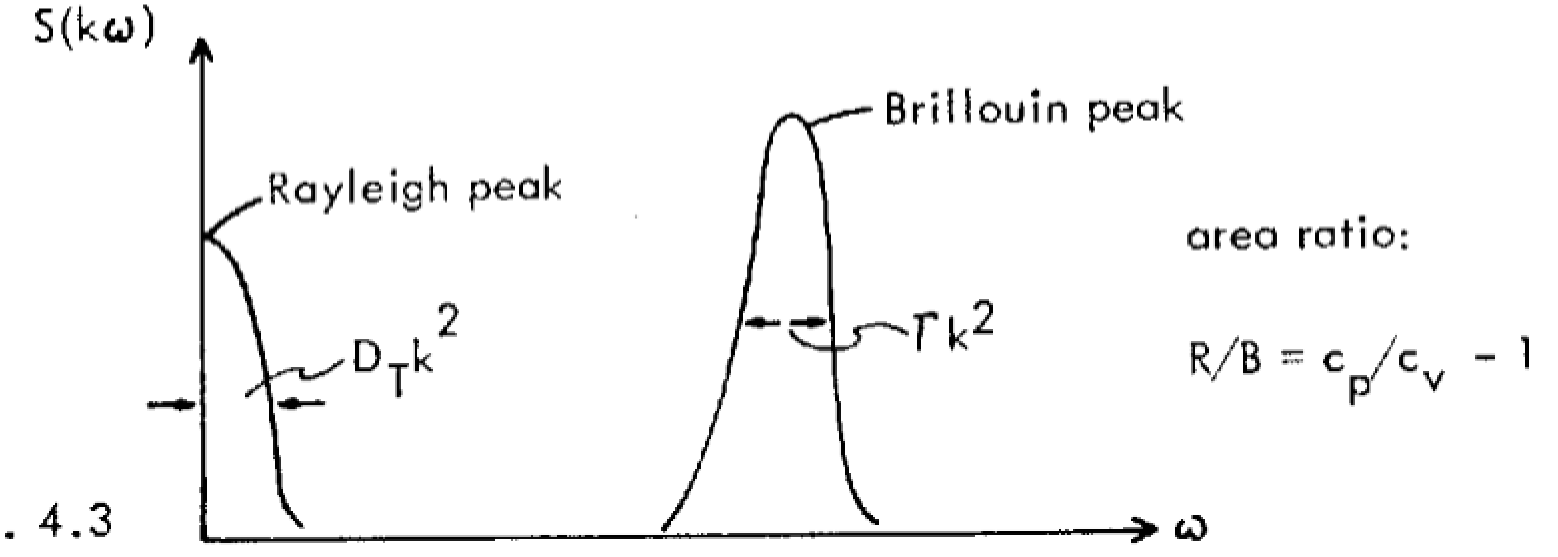
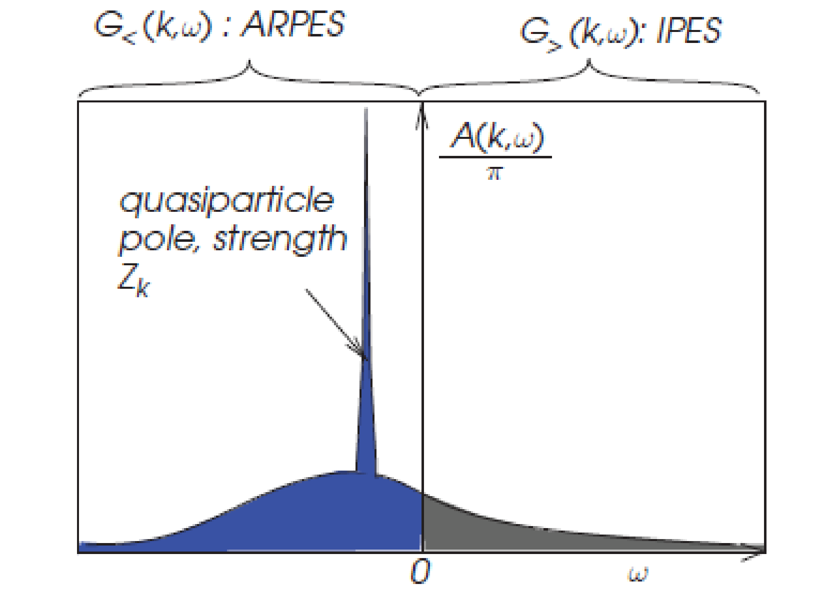
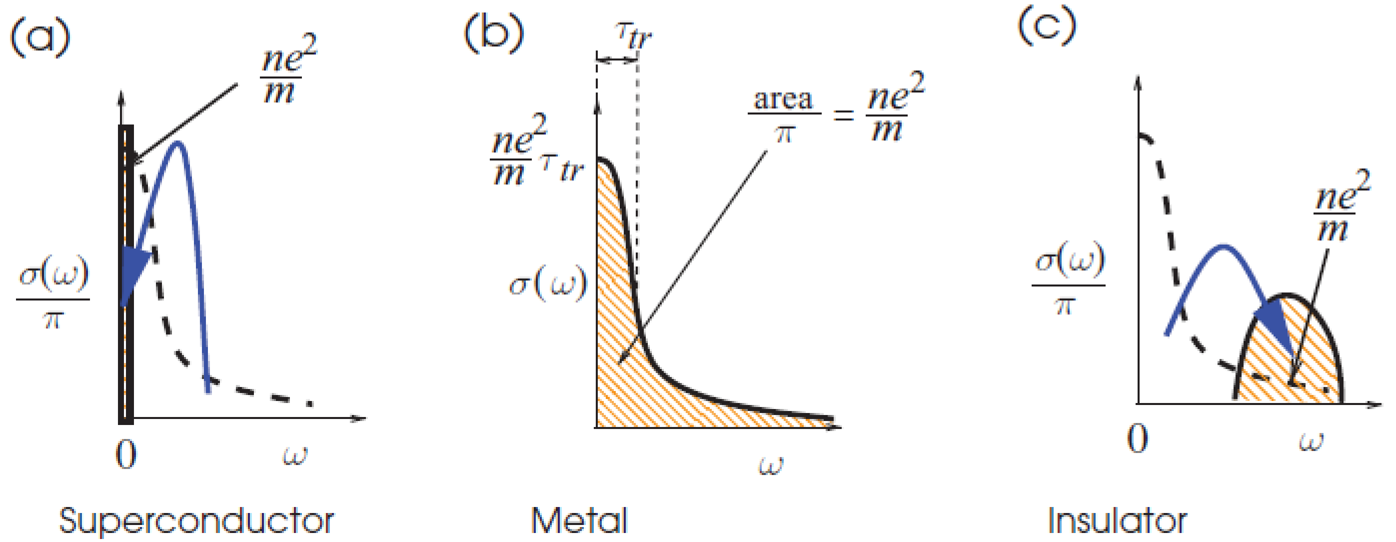


Fig. 4.3

Sum rules





The f-sum, rule illustrating (a) the spectral weight transfer down to the condensate in a superconductor; (b) the Drude weight in a simple metal; (c) the spectral weight transfer up to the conduction band in an insulator.

NAVIER-STOKES/2 FLUIDS EQUATIONS vs. QUANTUM HYDRODYNAMICS vs. (TD)DFT

$$-iM\omega j_i(\mathbf{r},\omega) = -\nabla_i[\delta p(\mathbf{r},\omega)\delta_{ij}] + \nabla_j[\delta\sigma_{ij}(\mathbf{r},\omega)]$$

$$-iM\omega v_{s,i}(\mathbf{r},\omega) = -\nabla_i \cdot [\delta\mu(\mathbf{r},\omega)\delta_{ij}] + \nabla_j \cdot [\delta\sigma^s_{ij}(\mathbf{r},\omega)],$$

$$\sigma_{ij} = \left(\eta(\omega) - \frac{p_0(n)}{i\omega}\right)\left(\frac{\partial v_{ni}}{\partial r_j} + \frac{\partial v_{nj}}{\partial r_i} - \frac{2}{3}\nabla\cdot\mathbf{v}_n\delta_{ij}\right) + \delta_{ij}[\zeta_2(\omega)\nabla\cdot\mathbf{v}_n + \zeta_1(\omega)\nabla\cdot\mathbf{j}_r]$$

$$\sigma^s_{ij} = \delta_{ij}[\zeta_3(\omega)\nabla\cdot\mathbf{j}_r + \zeta_4(\omega)\nabla\cdot\mathbf{v}_n]. \qquad (6.167)$$

$$\lim_{\omega\to 0}\lim_{k\to 0} Im[\frac{\omega}{k^2}\chi_{\mathbf{jj},L}(k,\omega)] = \frac{4}{3}\eta + \zeta_2$$

$$\lim_{\omega\to 0}\lim_{k\to 0} Im[\frac{\omega}{k^2}\chi_{\mathbf{j}\mathbf{v}_s}(k,\omega)] = \zeta_1 = \zeta_4$$

$$\lim_{\omega\to 0}\lim_{k\to 0} Im[\frac{\omega}{k^2}\chi_{\mathbf{v}_s\mathbf{v}_s}(k,\omega)] = \zeta_3$$

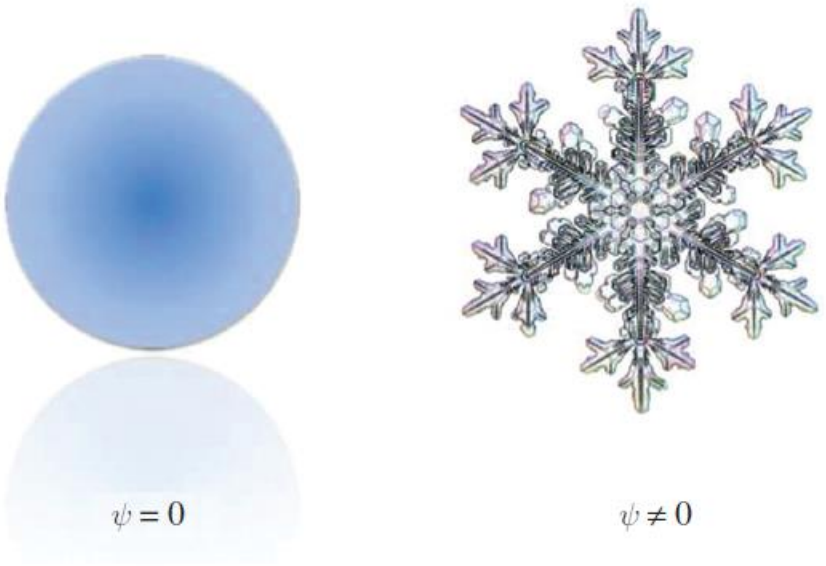
$$\lim_{\omega\to 0}\lim_{k\to 0} Im[\frac{\omega}{k^2}\chi_{\mathbf{jj},T}(k,\omega)] = \eta.$$

KUBO RELATIONS

ADDING BROKEN SYMMETRIES I

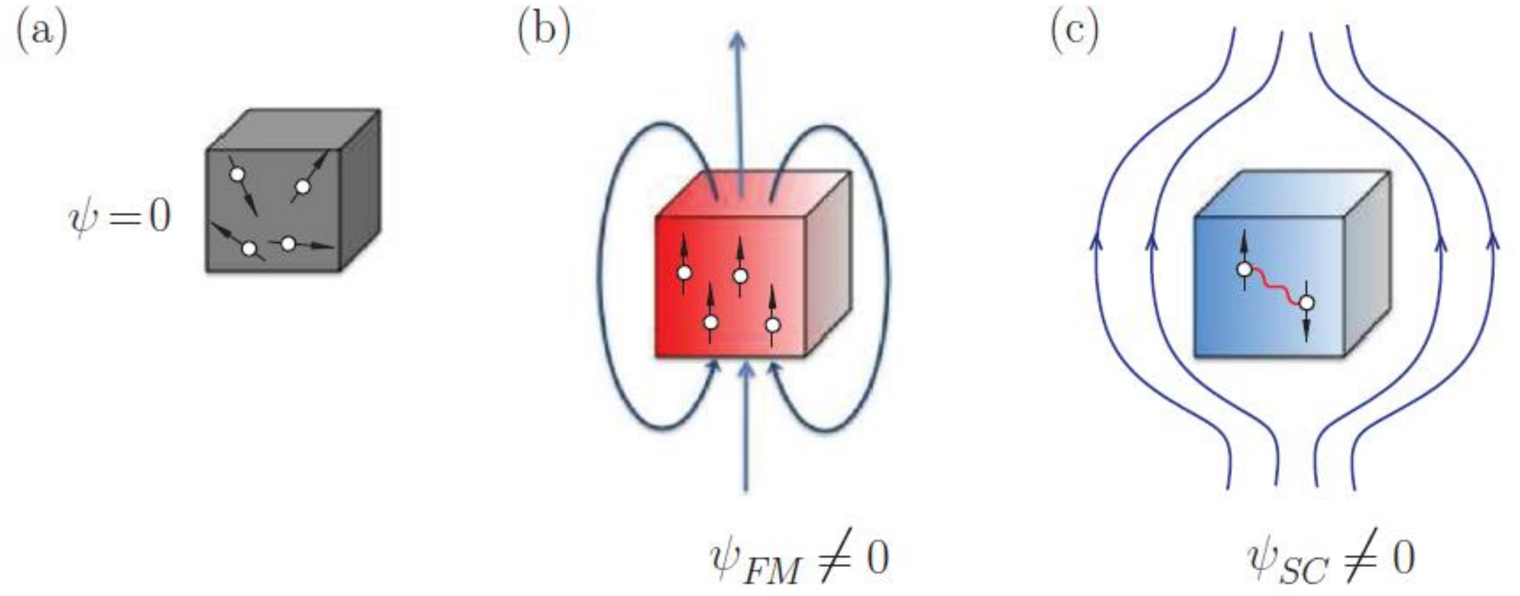
Table 11.1 Examples of Order parameters.

Order parameter	Realization	Microscopic origin
$m = \psi_1$	Ising ferromagnet	$\langle \hat{\sigma}_z \rangle$
$\psi = \psi_1 + i\psi_2$	Superfluid, superconductor	$\langle \hat{\psi}_B \rangle$, $\langle \hat{\psi}_\uparrow \hat{\psi}_\downarrow \rangle$
$\vec{M} = (\psi_1, \psi_2, \psi_3)$	Heisenberg ferromagnet	$\langle \vec{\sigma} \rangle$
$\Phi = \begin{pmatrix} \psi_1 + i\psi_2 \\ \psi_3 + i\psi_4 \end{pmatrix}$	Higgs field	$\begin{pmatrix} \langle \hat{\phi}_+ \rangle \\ \langle \hat{\phi}_- \rangle \end{pmatrix}$



Broken symmetry. The development of crystalline order within a spherical water droplet leads to the formation of a snowflake, reducing the symmetry from spherical to six-fold. Snowflake figure reproduced with permission from Kenneth G. Libbrecht.

ADDING BROKEN SYMMETRIES II



(a) In a normal metal, there is no long-range order. (b) Below the Curie temperature T_c of a ferromagnet, electron spins align to develop a ferromagnetic order parameter. The resulting metal has a finite magnetic moment. (c) Below the transition temperature of a superconductor, electrons pair together to develop a superconducting order parameter. The resulting metal exhibits the Meissner effect, excluding magnetic fields from its interior.

Superfluid «density» vs. «classical» Moment of Inertia

$$\mathcal{I}_{zz} = \int dr\,(r^2 - z^2)\rho_n.$$

$$\mathcal{I}_{ij} = \left[\frac{\partial \langle L_i\rangle_\omega}{\partial\omega_j}\right]_{\omega=0}.$$

$$\mathcal{I}_{zz} = i\int dr\;\varepsilon_{zsl}\varepsilon_{zml} r_s \left\{(\nabla_k)_m \left[e^{ik\cdot r}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\,\frac{\Upsilon^T(k,\omega)}{\omega}\right]\right\}_{k=0}.$$

$$\Upsilon^L(k,\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega(t-t')}\int dr\, e^{-ik\cdot(r-r')}\langle[g_s^L(rt), g_s^L(r't')]\rangle;$$

Table 11.2 Superconductivity and electroweak physics.

	Superconductivity	Electroweak
Order parameter	ψ	$\begin{pmatrix}\psi_0\\ \psi_1\end{pmatrix}$
	Pair condensate	Higgs condensate
Gauge field/symmetry	$(\phi, \mathbf{A})$	$\mathcal{A}_\mu = g' B_\mu + g(\vec{A}_\mu \cdot \vec{\tau})$
	$U(1)$	$U(1) \times SU(2)$
Penetration depth	$\lambda_L \sim 10^{-7}$ m	$\lambda_W \sim 10^{-18}$ m
Coherence length	$\xi = \frac{v_F}{\Delta} \sim 10^{-9}$–$10^{-7}$ m	$\xi_{EW} \sim 10^{-18}$ m
Condensation mechanism	Pairing	Unknown
Screened field	$\vec{B}$	$W^\pm$, Z
Massless gauge field	None	Electromagnetism A_μ

SUPERFLUIDITY vs FUNDAMENTAL INTERACTIONS e.g.: ELECTROWEAK PHYSICS I

SUPERFLUIDITY vs FUNDAMENTAL INTERACTIONS e.g.: ELECTROWEAK PHYSICS II

The basic physics of the standard model can be derived using the techniques of Ginzburg–Landau theory, by examining the interaction of the Higgs condensate with gauge fields. In its simplest version, first written down by Weinberg [2], this is given by (see Example 11.9)

$$S_\Psi = -\int d^4x \left[\frac{1}{2} |(\nabla_\mu - i\mathcal{A}_\mu)\Psi|^2 + \frac{u}{2}\left(\Psi^\dagger \Psi - 1\right)^2 \right], \qquad (11.121)$$

$$\mathcal{A}_\mu = g\vec{A}_\mu \cdot \vec{\tau} + g' B_\mu,$$

$$\mathcal{A}_\mu \rightarrow \begin{cases} Z,\ W^\pm & \text{neutral/charged vector bosons} \\ A. & \text{photon} \end{cases}$$

Evaluation is competences-based

- Specific competences

  Conceptual knowledge and how-to: up to 18 points

  Procedural knowledge and how-to: up to 6 points

  Phenomenological knowledge and how-to: up to 4 points

- Life-skills (awareness, autonomy, problem solving, communication,...): up to 5 points

Exam: oral

- Specific competences

  Cprepare a dissertationor presentation on a specific topic of interest, not (specifically) treated in the course: applications of concepts and methods learned during the course

Textbooks and papers

- **General:**

– Piers Coleman, *Introduction to Many-Body Physics*, Cambridge University Press (2015) [GENERAL]

– Kadanoff and Baym, *Quantum Statistical Mechanics*, Benjamin (1962) [RESPONSE&GREEN FUNCTIONS]

– Iadonisi, Cantele, and Chiofalo, *Introduction to Solid State Physics and Crystalline Nanostructures*, Springer (2014) [(TD)DFT+]

– Chaikin and Lubensky, *Principles of Condensed Matter Physics*, Cambridge University Press (1995) [EXOTIC STUFF]

– Grosso and Pastori Parravicini, *Solid State Physics*, Academic Press (2000) [QUANTUM SOLIDS]

- **Pedagogical reserach papers for specific parts:**

– Martin, *Measurements and Correlation Functions*, Gordon and Breach (1968)

– Vignale, Ullrich, Conti, *Time-Dependent DFT and beyond the Adiabatic Local Density Approximation*, PRL 79, 4878 (1997)

– Baym, *Microscopic Description of* Superfluidity, Math. Methods in *Solid-State&Superfluid Theory*, Clark&Derrick Eds., Oliver&Boyd (1969)

– Hohenberg and Martin, *Microscopic Theory of Superfluid Helium*, Annals of Physics 34, 291-359 (1965)

– Giamarchi, *Quantum Physics in One Dimension*, Oxford Science Pub. (2006)

– Foulkes, Mitas, Needs, and Rajagopal, Quantum Monte Carlo Simulations of Solids, Revue of Modern Physics 73, 33 (2001); Schollwok and White, *Methods for Time Dependence in DMRG*, in *Effective Models for Low-Dimensional Strongly Correlated Systems*, Batrouni and Poilblanc Eds., p. 155 AIP, Melville, New York (2006)

- **More**

– Nozières and Pines, *Theory of Quantum Liquids I – II*, Westview Press (1999)/Pines, *The Many-Body Problem*, Wiley (1997)

– Forster, *Hydrodynamic Fluctuations, Broken Symmetry, And Correlation Functions,* Adv. Books Classics (1995)

– Bloomfield, *How Things Work*, Wiley (2013)

MORE TOOLS AND TRAINING OPPORTUNITIES

- Connection with the course of Numerical Method (Prof. Massimo D'Elia): possibility of shared theoretical/simulational labs on specific problems of interest
- Seminars on Density Matrix Renormalization Group simulational method (Dr. Davide Rossini): to be organized
- First Conference on Quantum Gases, Fundamental Interactions, and Cosmology (October 2017)

THANK YOU FOR YOUR ATTENTION!