RESEARCH STATEMENT
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Concorso per un posto di professore associato settore scientifico-disciplinare FIS03
Universita' di Salerno
9 e 10 Marzo 2006

Outlines

- **General considerations on motivations, objectives, and methodological approach**

- **Relevant results of past research acvitivity**
  - **Theory and Phenomenology of High-Tc Superconductivity**
  - **Screening, Structure and Excitations in Charged Bose fluids**
  - **Time-Dependent Density Functional Theory of Superfluids**
  - **Coherent *vs.* Incoherent Dynamics of BECs**

- **More recent research: present projects and future plans**
  - **Resonant Superfluidity in Quantum Degenerate Fermi Atomic gases**
  - **Atomtronics with Quantum Resonant Tunneling-based Devices**
  - **Test of the Equivalence Principle with Quantum Degenerate Atomic Gases**

❑ Motivations: Ethics of the "what is it useful for?"

➢ General

✓ Research work, especially associated to mentoring, is a paradigm of individual and collective growth: *learn-how to (solve problems) especially through mistakes, recognize and exploit previous understanding, acquire new abilities, integrate with other's competences (team working), contribute to community advancement, learn to be autonomous, mentor youngers to become autonomous*

➢ Specific

✓ Experimental interest

✓ Need to develop theoretical tools, including those useful to improve communication among different disciplines

✓ Need for better integration of science and society to: increase general cultural level and motivate funding priorities in political agenda

❑ Objectives:

- Personal growth and development as researcher and as individual in a society

- Contribute to knowledge advancement, in connection with experiments

- Conjugate research and mentoring for mutual benefit

- Disseminate the results, also at non-specialist level

❑ Methodology

General considerations…

➢ Attitudes

- ✓ Stick to aforementioned motivations ("ethical" approach)
- ✓ Curiosity for innovative subjects and techniques
- ✓ Multidisciplinarity-oriented approach
- ✓ Diversification of experiences
- ✓ Working in team

➢ Acquisition and development of other competences and experiences besides knowledge advancement and technical skills:

- ✓ Methodologies for problem solving
- ✓ Working within (also different) groups and in intl. contexts
- ✓ Communication at both specialized level and popular-science level
- ✓ Mentoring of youngers using a motivational approach, teaching knowledge and know-how, stimulating to experience diversification
- ✓ Invent, manage and lead projects

Past research activity…

Theory and Phenomenology of High-Tc Superconductivity

[PRB93,PLA95, N.Cim.D96, PSSB03]

Within Idonisi's group@Napoli

Past research activity…

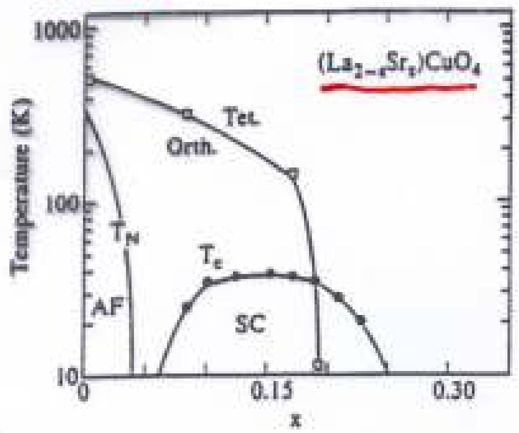
- **High-Tc superconductors have a very complex phase diagram, indicating that many different excitations (phonons, spin fluctuations,…) play a role in determining the normal and the superconducting state**

- **They can be classified in a sense as heavily doped polar semiconductors**

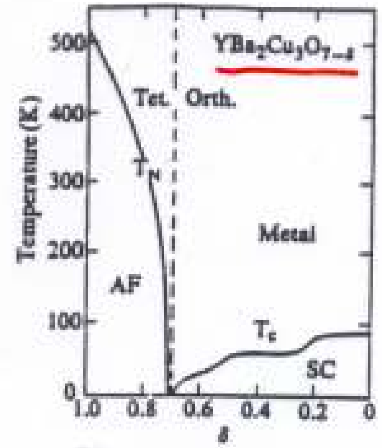
- **Optical phonons certainly play a role as well as the heavy doping, as normal and super-state properties depend on the density**

Past research activity…

Heavy doping!



a)



b)

x, δ connected to the carrier density

$$\frac{\varepsilon_\infty}{\varepsilon_0} = \prod_i^n \left(\frac{\omega_T^i}{\omega_L^i} \right)^2$$

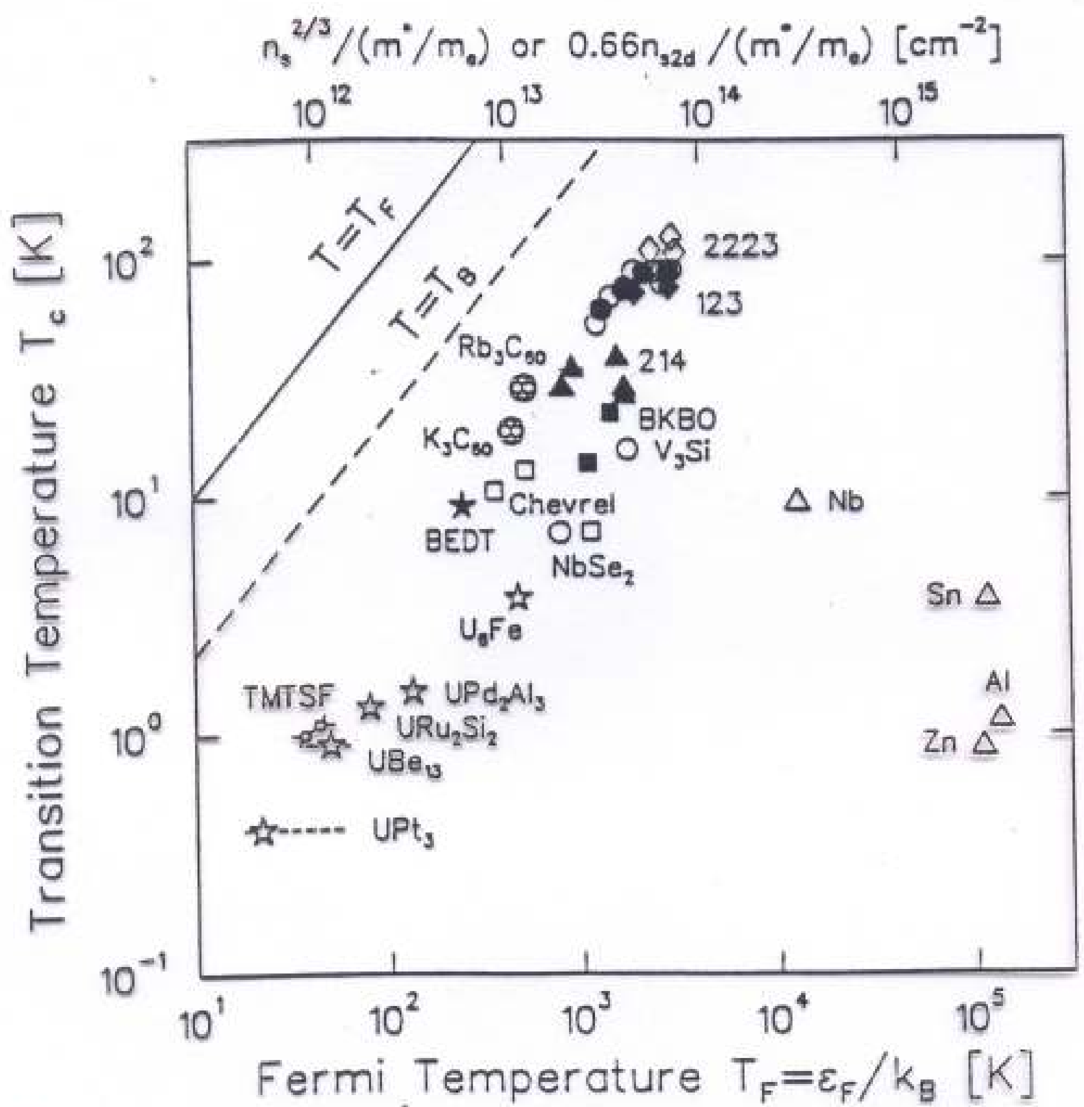
is **very small!**

(strong ionicity)

Mode	KK-analysis		Oscil. fit			Theory		
	ω_{TO}	ω_{LO}	ω_{TO}	γ_{TO}	S	ω_{TO}	ω_{LO}	S
A_{2u}	108	108	105	5	2.2	116	118	2.0
A_{2u}	156	187	145	12	3.0	153	163	3.3
A_{2u}	216	255	210	6	0.5	189	203	1.6
A_{2u}	370	466	365	20	2.1	390	455	1.7
A_{2u}	640	661	650	20	0.15	540	543	0.3
E_u	65	72				55	62	2.5
E_u	120	122	115	4	2.0	105	106	0.1
E_u	189	198	188	6	1.8	214	243	2.1
E_u	248	255	244	5	1.2	276	318	0.7
E_u	355	416	350	18	2.2	355	414	1.5
E_u	579	629	596	17	0.5	502	507	0.6

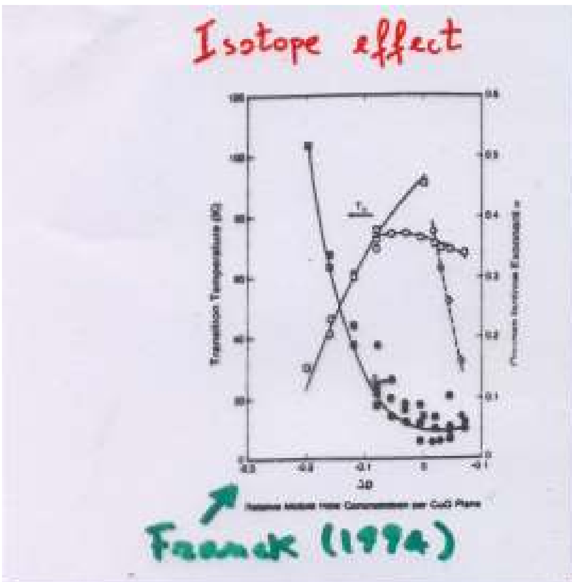
Oscillator strength ($\propto \omega_T, \omega_L$ splitting)

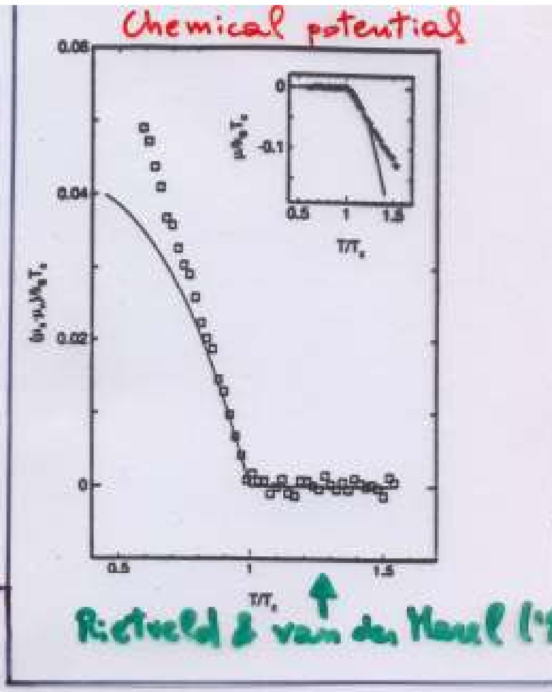
Past research activity…

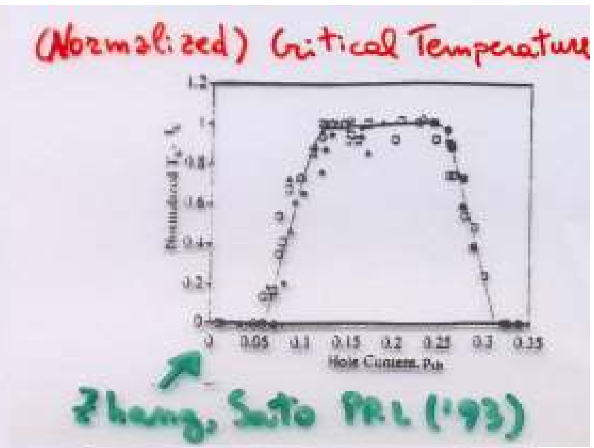


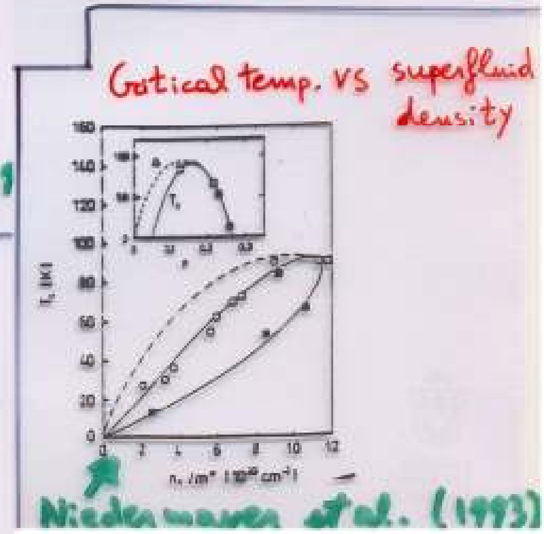
Uemura et al., PRL 66, 2665 ('91)

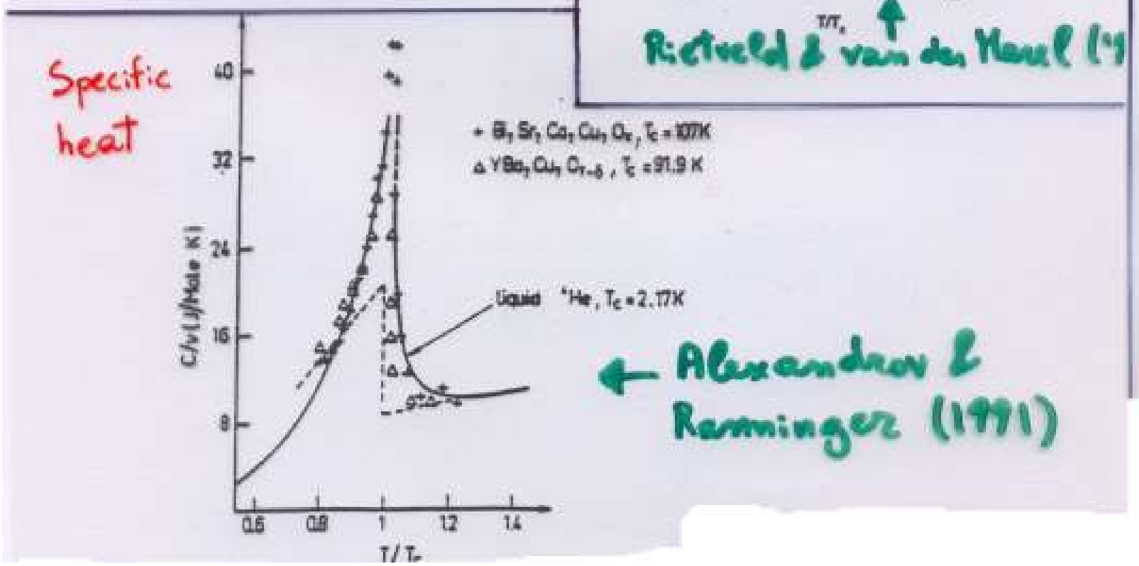
Past research activity…



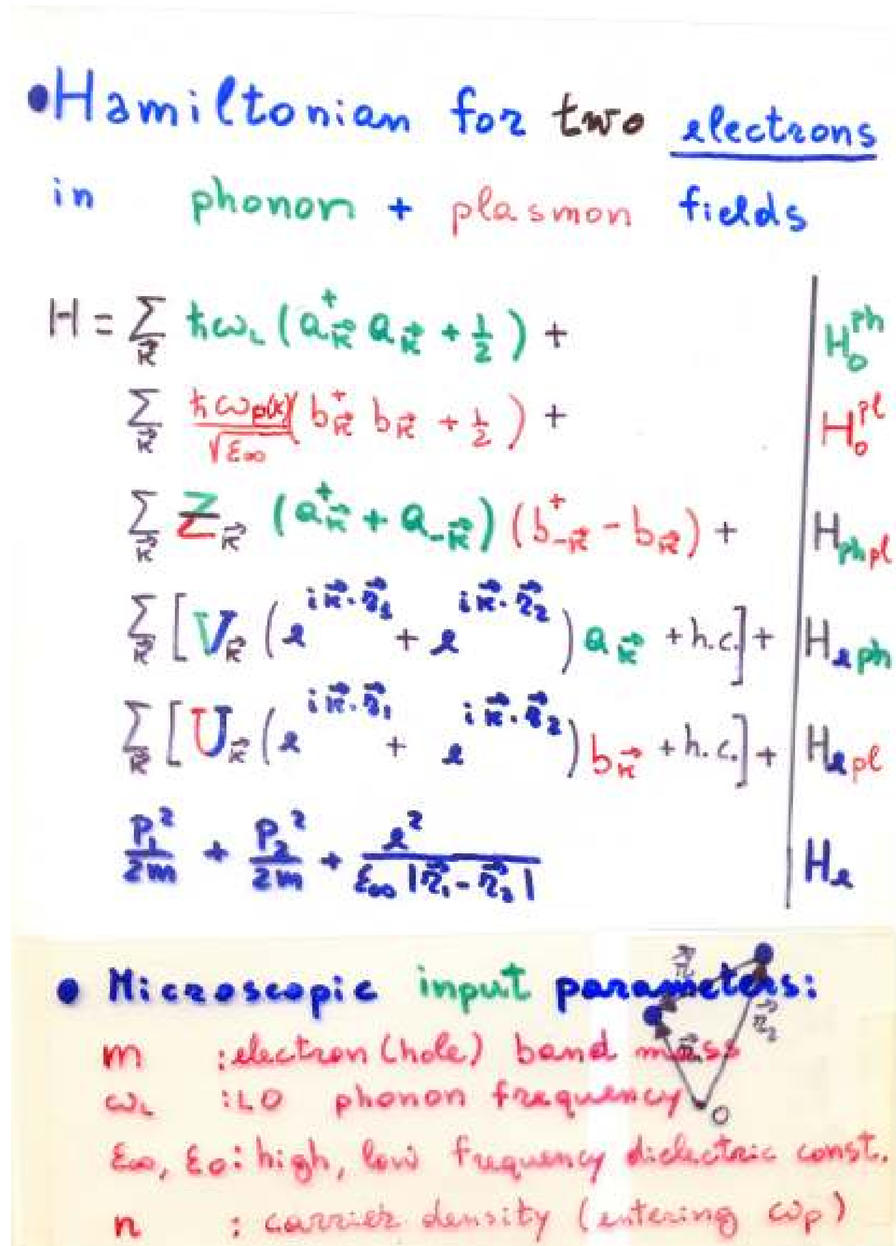








Past research activity…



➢ **Microscopic model with optical -phonon and plasmon cooperative effects at a variational +RPA level, yielding density-dependent pairing properties**

Past research activity…

α electron-phonon coupling

$\lambda = \omega_p / \omega_L \propto \sqrt{n}$

$\eta = \varepsilon_0 / \varepsilon_\infty$

Binding energy

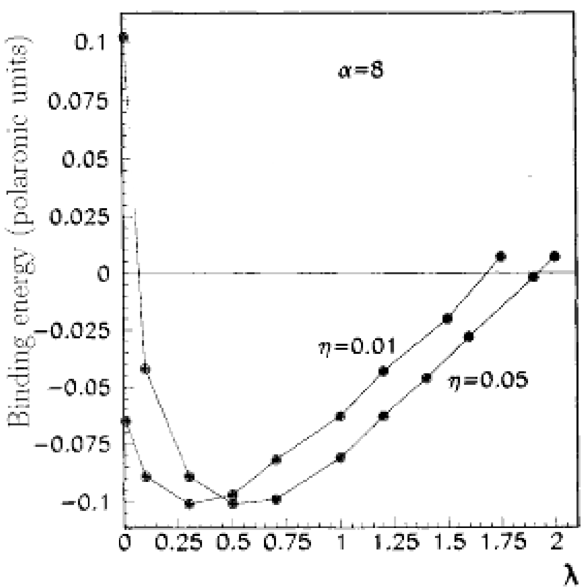


FIG. 2. Binding energies of the bi-plasma-polaron are shown as function of λ for $\alpha=8$ and $\eta=0.01$ and 0.05. $\lambda=1$ indicates an electronic density of 4.45×10^{19} cm^{-3}. The energies are in units of $\hbar\omega_l$.

Effective mass

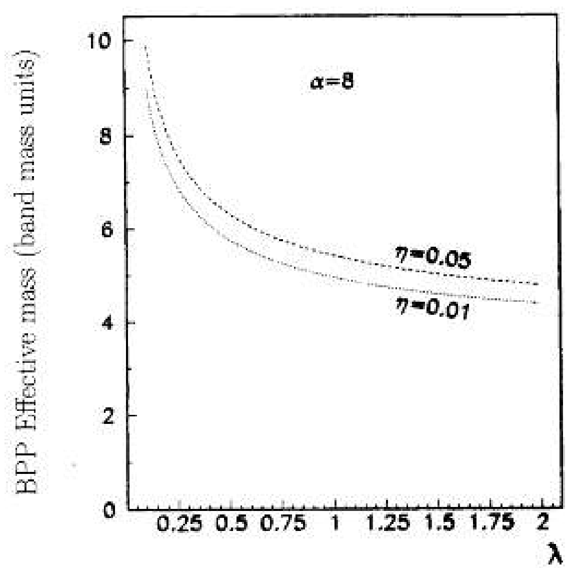
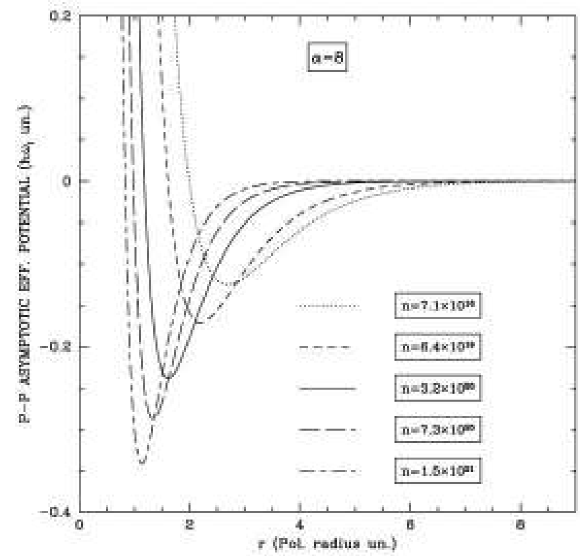
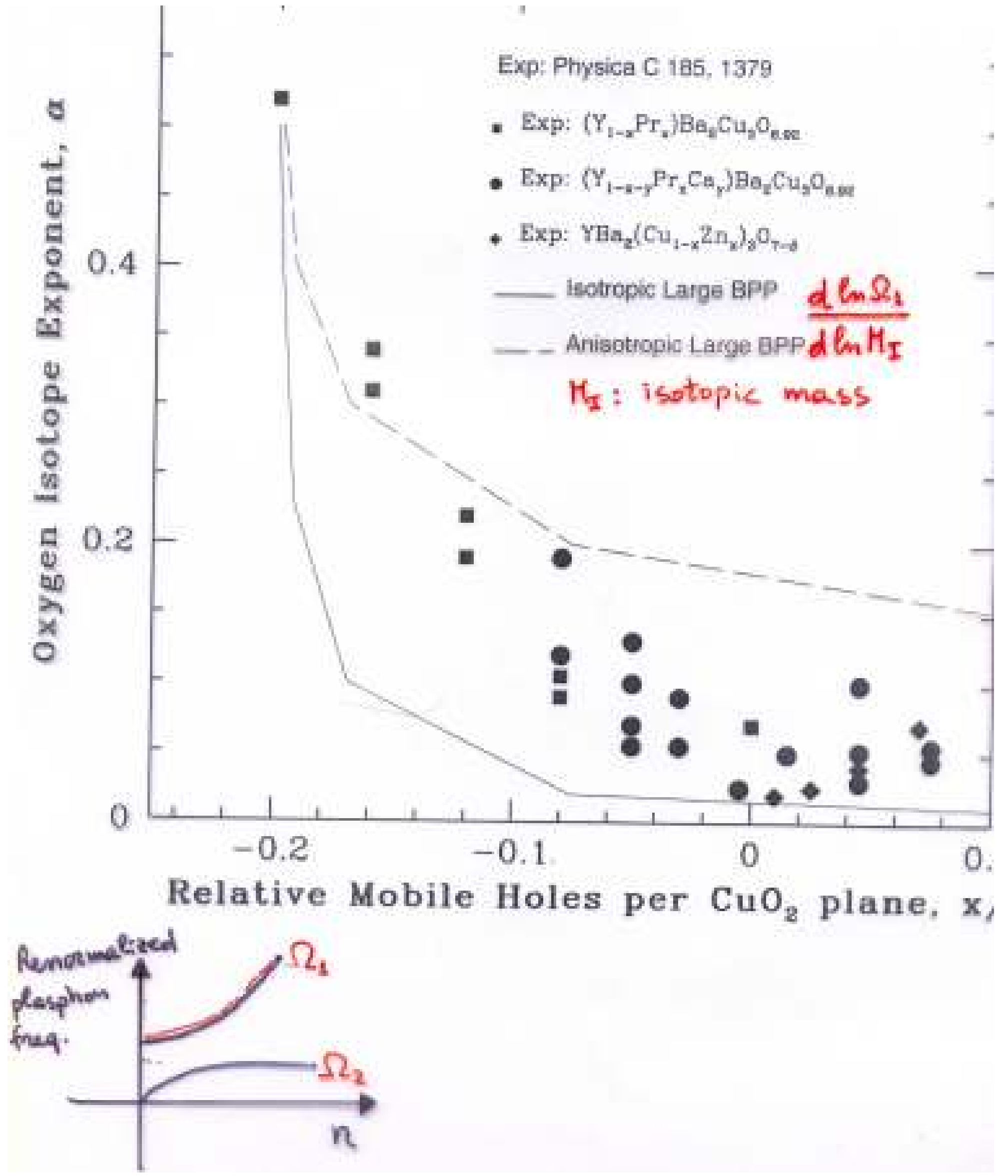


FIG. 4. Bi-plasma-polaron effective mass M^* are shown as function of λ for $\alpha=8$ and $\eta=0.01$ and 0.05.

Pseudopotential



Isotope effect



Past research activity…

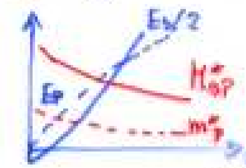
- **Determine Tc and thermodynamic properties from the boson-fermion model [Friedberg and TD Lee, Ranninger *et al.*] in which polarons and bipolarons coexist**

$$H = H_{pol} + H_{bipol} + H_{pol-bipol}$$

solved at 0-th loop level with density-dep. ingredients from microscopic model

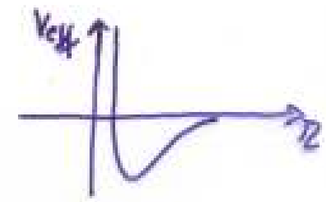
• Coexistence of BPs and Ps :

$$H = \sum_{\vec Q}\left(E_b(n) + \frac{\hbar^2 Q^2}{2M^*(n)} - 2\mu\right) b^\dagger_{\vec Q} b_{\vec Q} + \sum_{\vec q}\left(\frac{\hbar^2 q^2}{2m^*(n)} - \mu\right) c^\dagger_{\vec q} c_{\vec q} + \frac{1}{\sqrt{V}}\sum_{\vec Q,\vec q}\left(g(\vec Q,\vec q;n)\, b^\dagger_{\vec Q}\, c_{\frac{\vec Q}{2}+\vec q,\uparrow}\, c_{\frac{\vec Q}{2}-\vec q,\downarrow} + h.c.\right)$$



- $E_b(n)$, $M^*(n)$, $m^*(n)$ from microscopical model
- $g(\vec Q,\vec q;n) = g_0(n)\, e^{-\frac{(q-q_c)^2}{k_r^2}}$

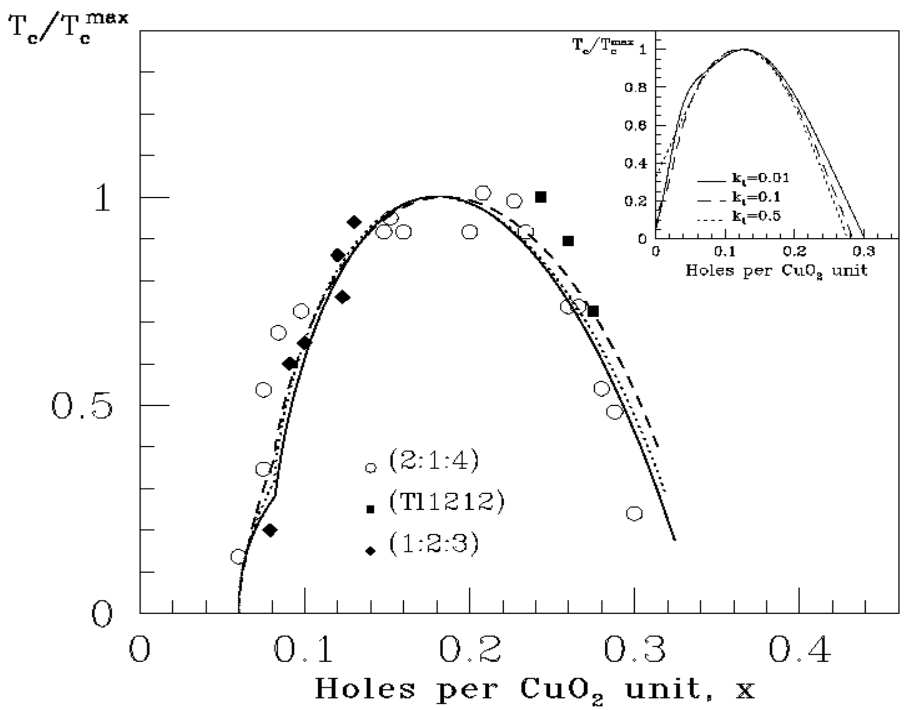
$$\begin{cases} q_c = \sqrt{2m^*\mu}/\hbar \\ k_r = \text{cutoff} \end{cases}$$

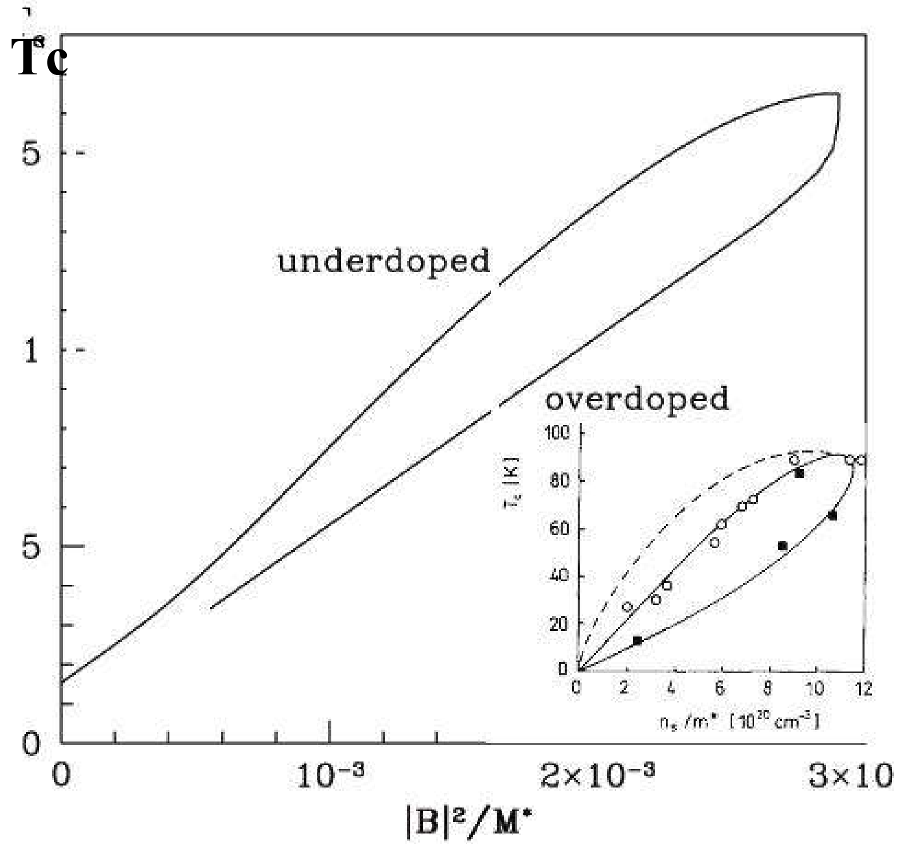


estimated from the polaron-polaron effective potential $V_{eff}(r)$

- $\mu_{BP} = 2\mu_P$

Past research activity…





Exp.: Zhang *et al.* PRL 1993

Exp.: Niedermayer *et al.* PRL 1993

Past research activity…

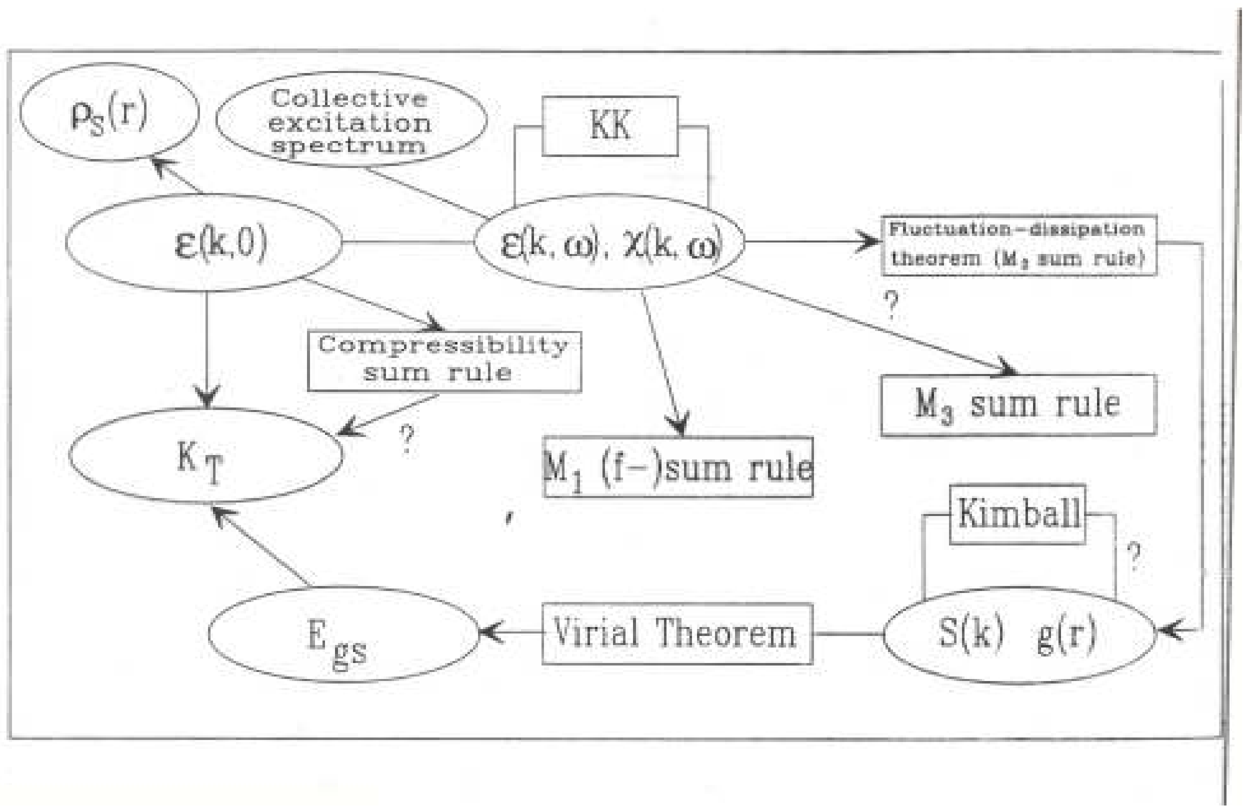
Screening, Structure, and Excitations in Charged Bose Fluids

[Mod.Phys.Lett.B94,JPCM94, JPCM95, JPCM96]

WithinTosi's group@SNS

➢ The ground state and the structure of Charged Bose Fluids may be relevant as a model system for HTcSC, neutron stars physics, and to disentangle the effects of statistics in the better known analogous Fermi (electron) systems

➢ Approach: use of static local-field theories for the dielectric function together with sum rules



➢ Response function

$$\chi(k,\omega) = \frac{\chi_0(k,\omega)}{1 - v_k[1 - G(k,\omega)\chi_0(k,\omega)]}$$

with approximations e.g.

$$G(k,\omega)\,;\quad G_{STLS}(k,\omega) = -\frac{1}{N}\sum_{\vec{k}'} \frac{\vec{k}\cdot\vec{k}'}{k'^2}[S(|\vec{k}-\vec{k}'|)-1]$$

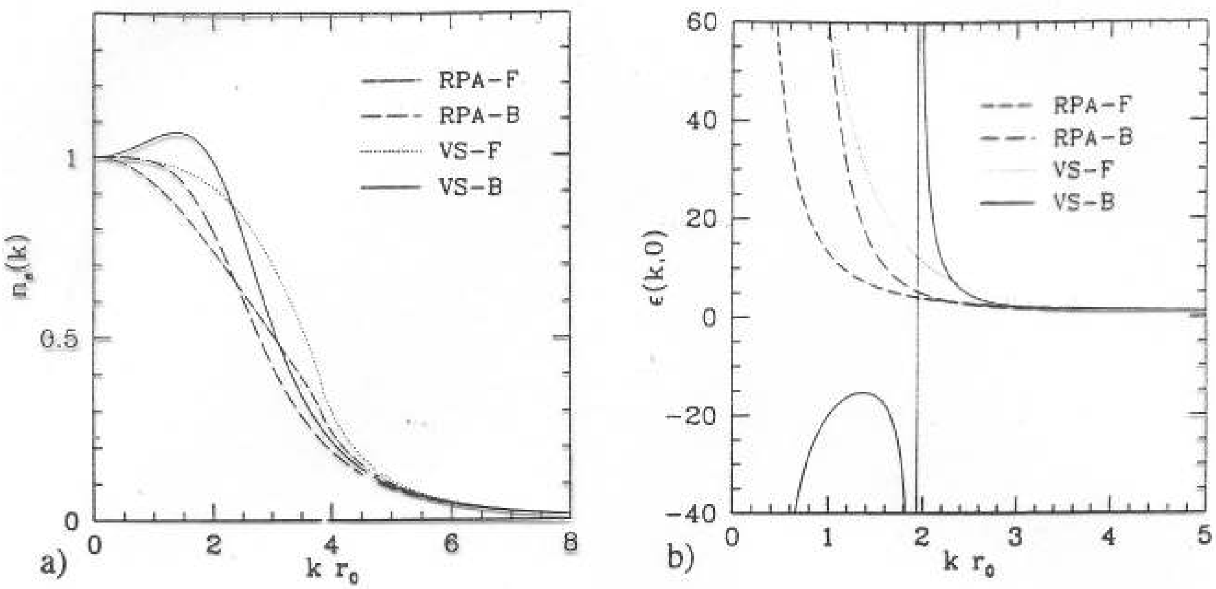
and subsequent improvements (VS satisfying compressibility sum-rule, PV satisfying 3-rd moment)

➢ Correlation energy

In comparison with QMC data, STLS, and VS and PV are accurate in predicting the gs-energy, VS more suitable at large r_s. No appreciable differences between gs-energy of bosons and exch+correlation energy of fermions above r_s about 20. STLS only satisfies Kimball-Niklasson's relation at r=0. Only VS accounts for the static dielectric function

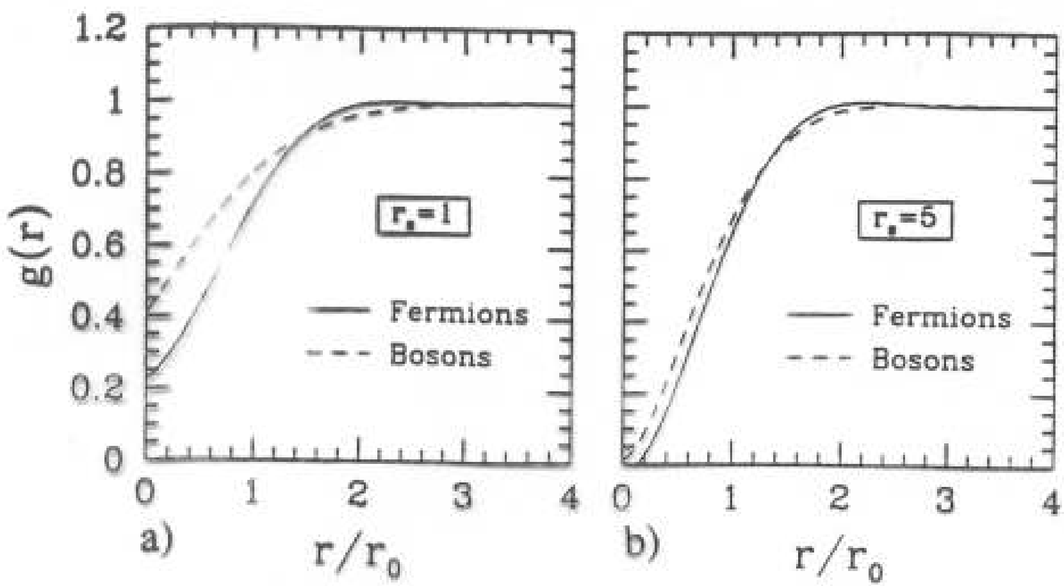
➢ Screening

Bosons tend to overscreen (negative dielectric function) due to local piling up



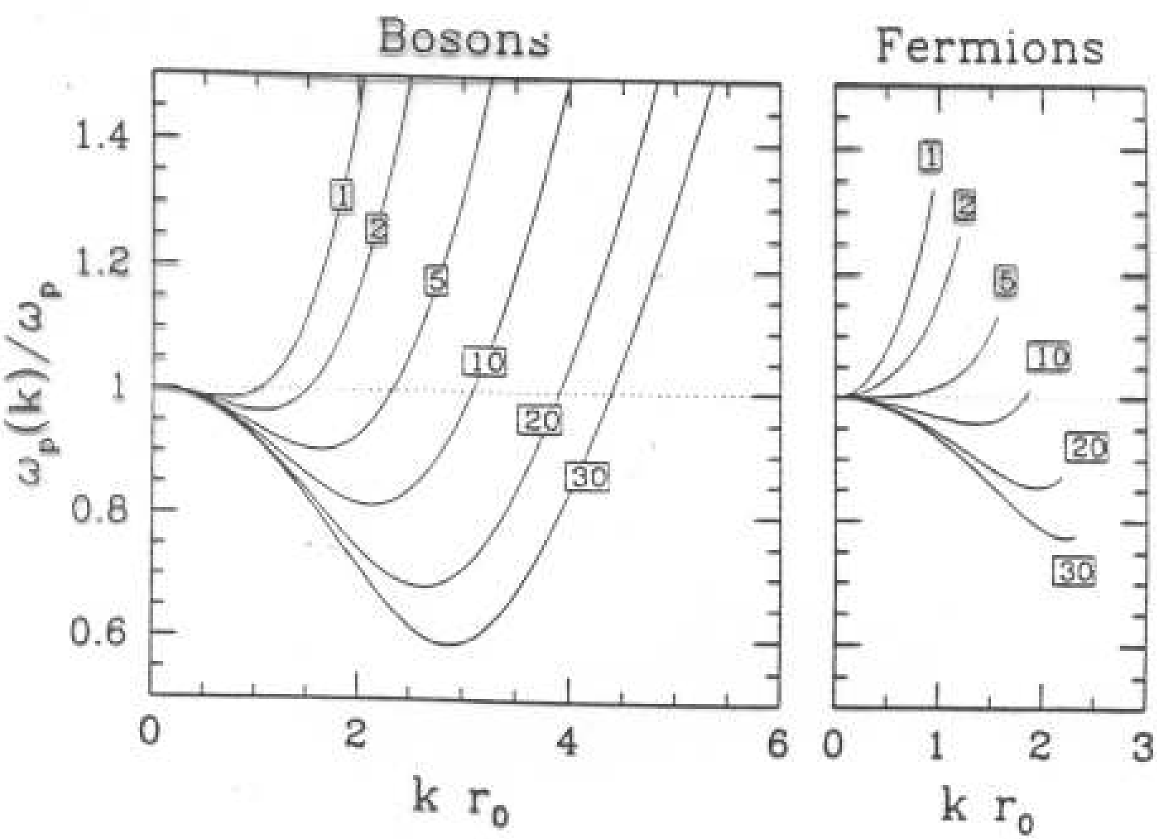
➢ Structure

Differences between bosons and fermions at short distance and low coupling due to statistics



➢ Excitations

Bosons have negative dispersion at k->0 at all r_s values (E_{kin}=0 and strong correlations, see also negative dielectric function). Plasmon exhausts the density-density sum rule



Past research activity…

Time-Dependent Density Functional Theory For Superfluids

[PhysicaB98,EPL01]

In collaboration with M. Tosi@SNS

Past research activity…

- Bose-Einstein Condensation in alkali gases has become an ideal laboratory for condensed matter physics, where a inhomogeneous (e.g. trapped) superfluid is made available in the presence of (tunable) interactions and under non-equilibrium conditions

- Collective excitations in both the collisional and collisionless regimes, propagation of sound waves, transport behaviour for atom-optical applications have become experimentally available

- Need for a theory of inhomogeneous Bose Fluids at finite T, capable of spanning the whole region from the collisionless to the hydrodynamic regimes

➢ **Density Functional Theory is a possible approach to inhomogeneous systems. It is based on:**

✓ Hohenberg-Kohn theorem: properties of interacting inhomo system in one-to-one correspondence with the density

✓ Kohn-Sham scheme: density calculated from the Schroedinger eq. for appropriate single-particle fictitious orbitals in external effective potential $V_{eff} = V_e + V_{Hartree} + V_{xc}$

✓ V_{eff} has to be approximated: *e.g* from xc-energy of the homo system at the local density of the inhomo one (Local Density Approx)

➢ **Time-Dependent Density Functional Theory:**

✓ Hohenberg-Kohn theorem: holds provided n is known at t=0

✓ Kohn-Sham scheme: similarly applies

✓ V_{eff} has to be approximated : LDA fails since the xc-potential is nonlocal! Thus, TD-DFT expressed in terms of current [Vignale&Kohn]

- **Current fluctuation, effective vector potential and KS response**

$$\delta J_i(r,\omega) = \int dr' \chi_{ij}^{KS}(r,r',\omega) A_{eff,j}(r',\omega)$$

$$A_{eff} = A_e + A_{Hartree} + A_{xc}$$

$$\chi_{ij}^{KS}(r,r',\omega) = \frac{n_0(r)}{m}\delta(r-r')\delta_{ij} \quad + \quad \chi_{ij}^{RPA}(r,r',\omega)$$

Generalized hydro with complex and freq.-dependent viscosities (collisional)

Landau damping (collisionless)

- **Correlations beyond mean-field are contained into**

$$A_{xc,i}(r,\omega) = \int dr' f_{xc,ij}(r,r',\omega) \delta J_j(r',\omega)$$

and the f_{xc} kernels are determined from the homo system through LDA at the local super- and normal-fluid densities

➢ Landau's equations (homo system)

$$\frac{\partial \vec{J}}{\partial t} = -\vec{\nabla} \cdot \overleftrightarrow{T} = -\vec{\nabla} \cdot \left[p\delta_{ij} - \delta_{ij}\left(\zeta_2 \vec{\nabla} \cdot \vec{v}_n + \zeta_1 \vec{\nabla} \cdot \rho_s (\vec{v}_s - \vec{v}_n)\right) - \eta\left(\nabla_i \cdot v_{nj} + \nabla_j \cdot v_{ni} - \frac{2}{3}\delta_{ij} \vec{\nabla} \cdot \vec{v}_n\right)\right]$$

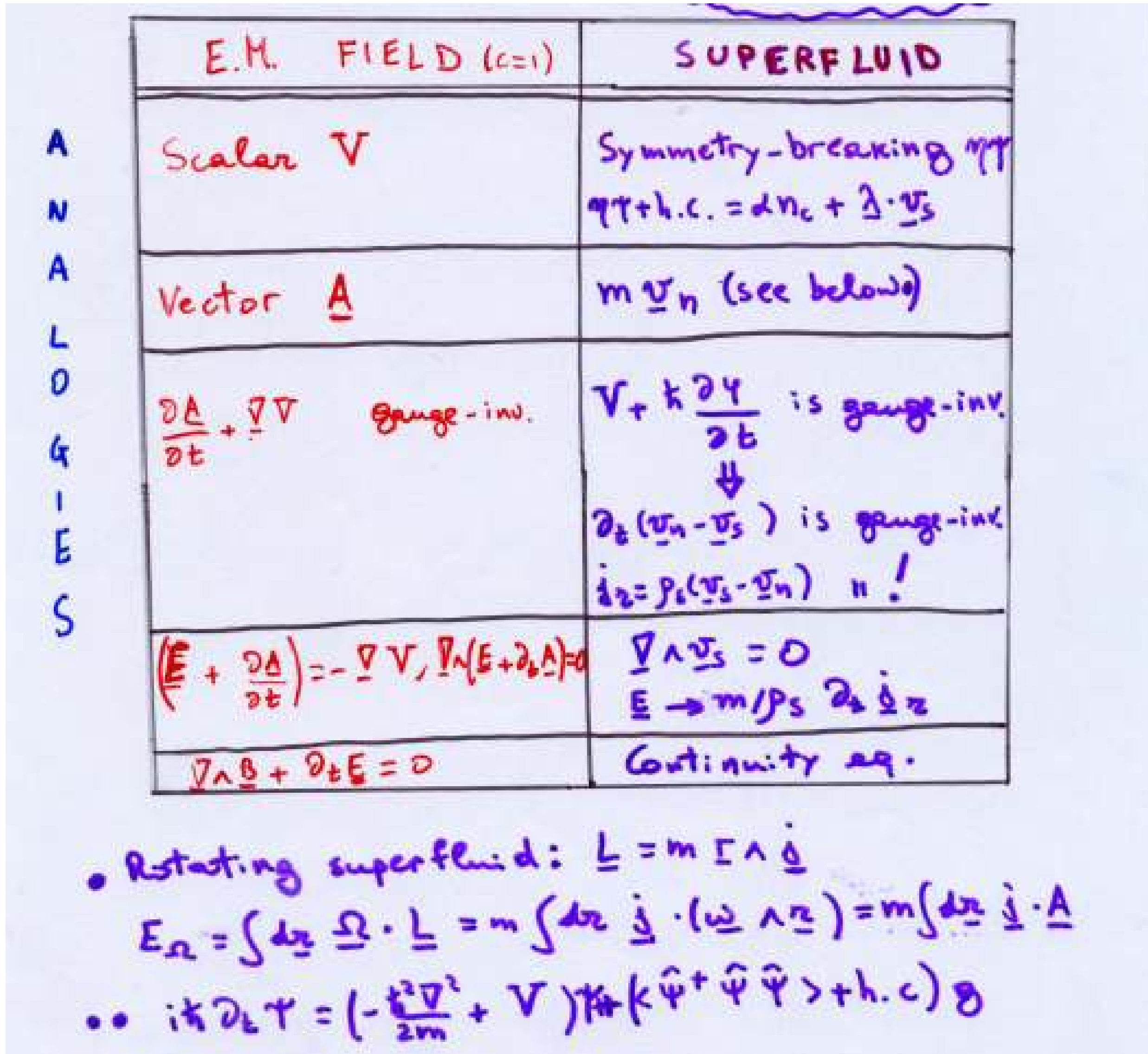
$$\frac{\partial \vec{v}_s}{\partial t} = -\vec{\nabla}\mu^{loc} = -\vec{\nabla}\left[\mu - \zeta_3 \vec{\nabla} \cdot \rho_s (\vec{v}_s - \vec{v}_n) - \zeta_4 \vec{\nabla} \cdot \vec{v}_n\right]$$

- ❖ **Galileian invariance holds**
- ❖ **The variables are: total current J and superfluid velocity v_s**
- ❖ **The 0-force and 0-torque theorems dictate that dJ/dt must be driven by the divergence of a symmetric tensor of 2nd rank**
- ❖ **v_s is irrotational in the absence of vortices thus dv_s/dt must be the gradient of a scalar quantity**
- ❖ **The internal driving forces are determined by v_n and the interdiffusion current $\rho_s (v_{s-} v_n)$**

➢ Extension to inhomo system and finite frequency

❖ **First step:**
Identify scalar and vector potentials, currents and driving forces

❖ **Finite frequency:**
Use of memory function formalism



ANALOGIES

E.M. FIELD (c=1)	SUPERFLUID
Scalar V	Symmetry-breaking $\psi\psi$ $\psi\psi$+h.c. = $d n_c$ + $\lambda \cdot v_s$
Vector $\underline{A}$	$m \underline{v}_n$ (see below)
$\frac{\partial \underline{A}}{\partial t} + \underline{\nabla} V$ gauge-inv.	$V + \hbar \frac{\partial \varphi}{\partial t}$ is gauge-inv. ⇓ $\partial_t(\underline{v}_n - \underline{v}_s)$ is gauge-inv. $\underline{j}_2 = \rho_s(\underline{v}_s - \underline{v}_n)$ " !
$\left(\underline{E} + \frac{\partial \underline{A}}{\partial t}\right) = -\underline{\nabla} V$, $\underline{\nabla} \wedge (\underline{E} + \partial_t \underline{A}) = 0$	$\underline{\nabla} \wedge \underline{v}_s = 0$ $\underline{E} \to m/\rho_s \, \partial_t \dot{\underline{j}}_2$
$\underline{\nabla} \wedge \underline{B} + \partial_t \underline{E} = 0$	Continuity eq.

• Rotating superfluid: $\underline{L} = m \, \underline{r} \wedge \dot{\underline{j}}$

$E_\Omega = \int d\underline{r} \, \underline{\Omega} \cdot \underline{L} = m \int d\underline{r} \, \underline{j} \cdot (\underline{\omega} \wedge \underline{r}) = m \int d\underline{r} \, \underline{j} \cdot \underline{A}$

•• $i\hbar \partial_t \Psi = \left(-\frac{\hbar^2 \nabla^2}{2m} + V\right) \Psi + \left(\langle \hat{\psi}^+ \hat{\psi} \hat{\psi} \rangle + h.c.\right) g$

➢ Extension to inhomo system and finite frequency

Use Ward identity to relate the effect of weak inhomo on the xc-kernels to their density dependence: compare the k->0 inhomo response functions to 1st order in the inhomogeneity with those of the homo system after switching on a density modulation $\delta\rho_\alpha(\vec{r}) = 2\xi_\alpha\bar{\rho}_\alpha\cos(\vec{q}\cdot\vec{r}),\quad \xi_\alpha = 1$

$$\lim_{q\to 0}\overleftrightarrow{f}_{\alpha\beta}^{\,inhom}(\vec{k}+\vec{q},\vec{k},\omega;\{\bar{\rho}_\alpha\}) = \sum_\gamma \xi_\gamma\bar{\rho}_\gamma\frac{\partial}{\partial\bar{\rho}_\gamma}\overleftrightarrow{f}_{\alpha\beta}^{\,hom}(\vec{k},\omega;\{\bar{\rho}_\alpha\})$$

$$\overleftrightarrow{f}_{\alpha\beta}^{\,inhom}(\vec{r},\vec{r}\,',\omega) = \overleftrightarrow{f}_{\alpha\beta}^{\,hom}(\vec{k},\omega;\{\bar{\rho}_\alpha(\vec{r})\})$$

Recipe

➢ Ingredients

❖ **Microscopic expressions for local** $\rho_s^{eq}(r)$ and $\rho_n^{eq}(r)$

❖ **xc Kernels** $\check{f}_{\alpha\beta}^{\mathrm{hom}}(k,\omega;\rho_s^{eq}(r),\rho_n^{eq}(r))$

➢ Preparation

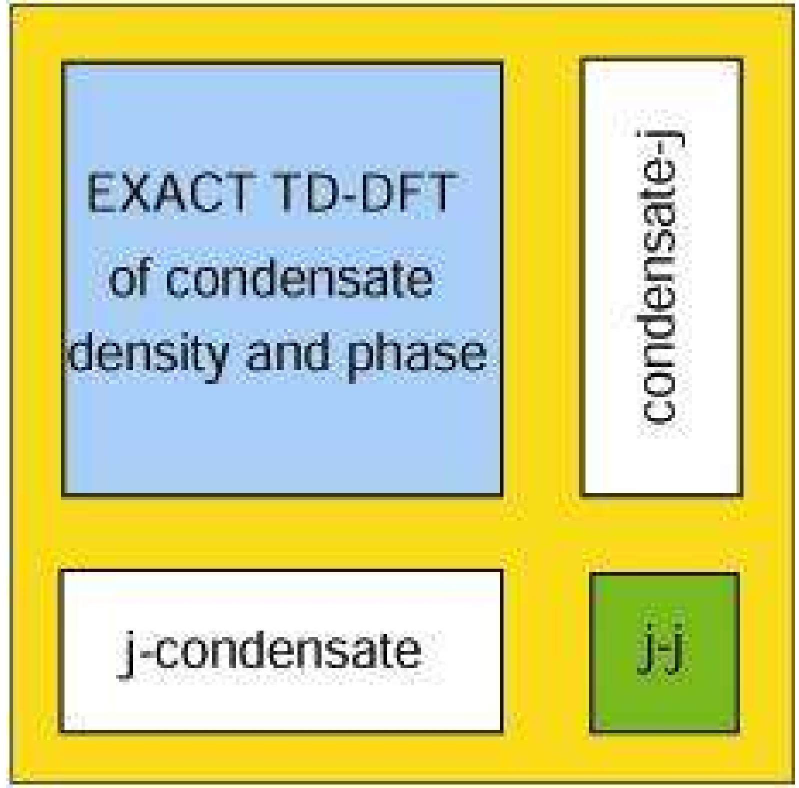
❖ **Evaluate** $\rho_s, \rho_n, \check{f}_{\alpha\beta}^{\mathrm{hom}}$ **(after QMC or perturbative methods)**

❖ **Relate viscoelastic spectra** $\zeta_i(\omega), \eta(\omega)$ to $\mathrm{f}_{\alpha\beta}$

❖ **Put everything into generalized Landau eqs. at finite frequency and solve**

Past research activity…

➢ **Identification of the ingredients: from the microscopic equations of motion for the currents**



❖ **Super and Normal densities**

$$\lim_{q_{rel}\to 0} \vec{\nabla}_1 \rho_s(1,2) = \vec{\nabla}_2 \frac{\delta \vec{J}(1)}{\delta \vec{v}_s(2)} \Big|_{\vec{A}} = \vec{\nabla}_R \rho_s(R)$$

$$\rho_n(r) = \rho(r) - \rho_s(r) \qquad\qquad 1 \equiv (r_1, t_1)$$

❖ Relation between the viscoelastic functions and the xc-kernels

$$\overset{t}{f}{}_{\alpha\beta}^{\,hom}(\omega) = \lim_{k\to 0} \frac{\omega^2}{k^2}\rho_\alpha \rho_\beta [\chi_{v_\alpha v_\beta}(k,\omega) - \chi^0_{v_\alpha v_\beta}(k,\omega)] \qquad \alpha, \beta \equiv s, n$$

- **Generalized Kubo formulae:**

$$\mathrm{Re}\left[\zeta_2(\omega) + \tfrac{4}{3}\eta(\omega)\right] = \lim_{k\to 0} -\frac{\omega m^2}{k^2}\, \mathrm{Im}\, \chi^L_{jj}(k,\omega)$$

$$\mathrm{Re}\left[\eta(\omega)\right] = \lim_{k\to 0} -\frac{\omega m^2}{k^2}\, \mathrm{Im}\, \chi^T_{jj}(k,\omega)$$

$$\mathrm{Re}\left[\zeta_3(\omega)\right] = \lim_{k\to 0} -\frac{\omega}{k^2}\, \mathrm{Im}\, \chi_{v_s v_s}(k,\omega)$$

$$\mathrm{Re}\left[\zeta_1(\omega)\right] = \mathrm{Re}\left[\zeta_4(\omega)\right] = \lim_{k\to 0} -\frac{\omega m}{k^2}\, \mathrm{Im}\, \chi^L_{j v_s}(k,\omega)$$

yielding the frequency-dependent visco-elastic coefficients $\eta(\omega)$, $\zeta_i(\omega)$

❖ And eventually….

$$\zeta_1(\omega;n,T) = -\frac{1}{i\omega}\left[f^{L}_{j\sigma_s}(\omega;\{\rho_\alpha\}) - \frac{\partial P_{ex}(n,T)}{\partial n}\Big|_T \right]$$

$$\zeta_2(\omega;n,T) = -\frac{1}{i\omega}\left[f^{L}_{jj}(\omega;\{\rho_\alpha\}) - \frac{4}{3} f^{T}_{jj}(\omega;\{\rho_\alpha\}) - n\frac{\partial P_{ex}}{\partial n}\Big|_T \right]$$

$$\zeta_3(\omega;n,T) = -\frac{1}{i\omega}\left[f_{\sigma_s\sigma_s}(\omega;\{\rho_\alpha\}) - \frac{\partial \mu_{ex}(n,T)}{\partial n}\Big|_T - \frac{TS}{C_v}\frac{\partial \mu_{ex}(n,T)}{\partial T}\Big|_n \right]$$

$$\zeta_4(\omega;n,T) = -\frac{1}{i\omega}\left[f^{L}_{\sigma_s j}(\omega;\{\rho_\alpha\}) - \frac{\partial P_{ex}(n,T)}{\partial n}\Big|_T \right]$$

$$\eta(\omega;n,T) = -\frac{1}{i\omega} f^{T}_{jj}(\omega;\{\rho_\alpha\})$$

Past research activity…

Coherent *vs.* Incoherent Dynamics of BECs

- ❑ Extracting physical quantities from BEC experiment
- ❑ 1D Bosons as a model for noninteracting 1D Fermions
- ❑ BECs in optical lattices: energy bands, dissipation of superfluidity, Josephson arrays, decoherence, and chaos

[PRL97,PLA99,PLA00,EPJD00,PRA01,PRL01,JPB01,EPL01, PRA01,JLTP02,PRE00,PRE00,PLA02,PRL02,JCP02]

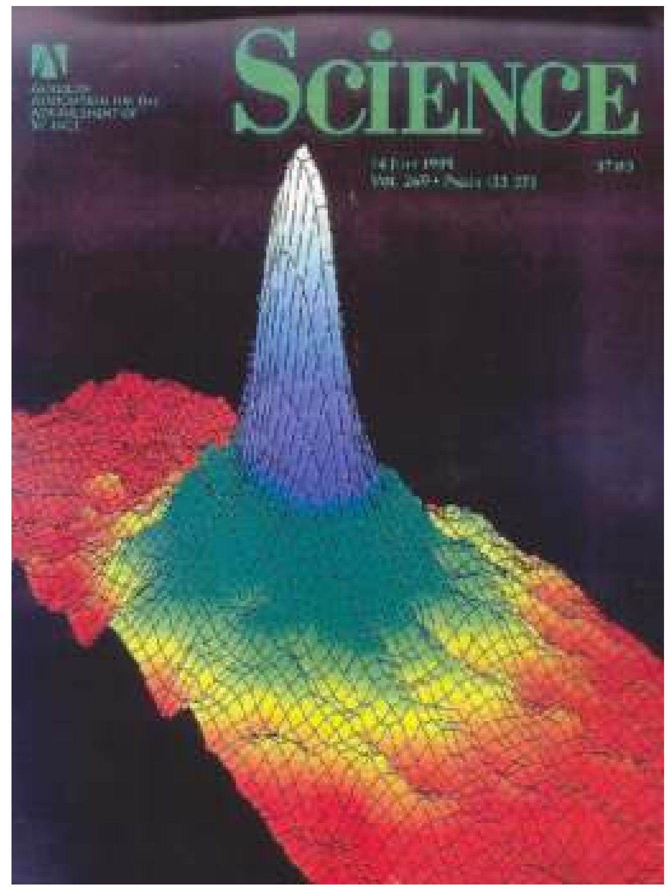
In collaboration with: M. Holland'group@JILA, E.Cornell's group@JILA, M. Tosi's group@SNS, M. Inguscio's group@LENS, S.Succi@IAC-Rome, F.Pistella&M.M. Cerimele@IAC-Rome

Past research activity…

- The very original idea of Bose and Einstein was first realized in 1995 at JILA in the groups of Cornell and Wieman and MIT in the group of Ketterle, after cooling trapped ^{87}Rb and ^{23}Na atomic gases down to nK temperatures below the threshold for BEC

- The vapour was kept in metastable state (dilute gas) against the formation of drops. The required low T and high n were obtained after using a combined technique of laser cooling, magnetic trapping and evaporative cooling

- Opened terrific perspectives for fundamental physics and applications under highly controllable conditions, with nonlinearities from tunable interactions and external drivings



- This has opened terrific perspectives for fundamental physics and applications under highly controllable conditions, since ultracold atomic gases:

- can be driven by tunable intrinsic (e.g. atomic interactions) and/or extrincsic (e.g. external fields) nonlinearities

- their chracterizing dimensionality, interaction strength (making them also noninteracting, a=0, by Feshbach resonance mechanism) and temperature are separately tunable

- their quantum state can be manipulated and addressed with high precision: from coherent to squeezed to topological states, in the future may be also Schroedinger-cat states,…

❑ **One of the first basic questions was: how to extract the physical quantities from the primary exp information, that is the optical imaging (absorption or dispersion) and thus the density profile after switching off the trap and expansion?**

➢ **Temperature: fit from the wings of the distribution (Maxwell-Boltzmann, noncondensed gas)**

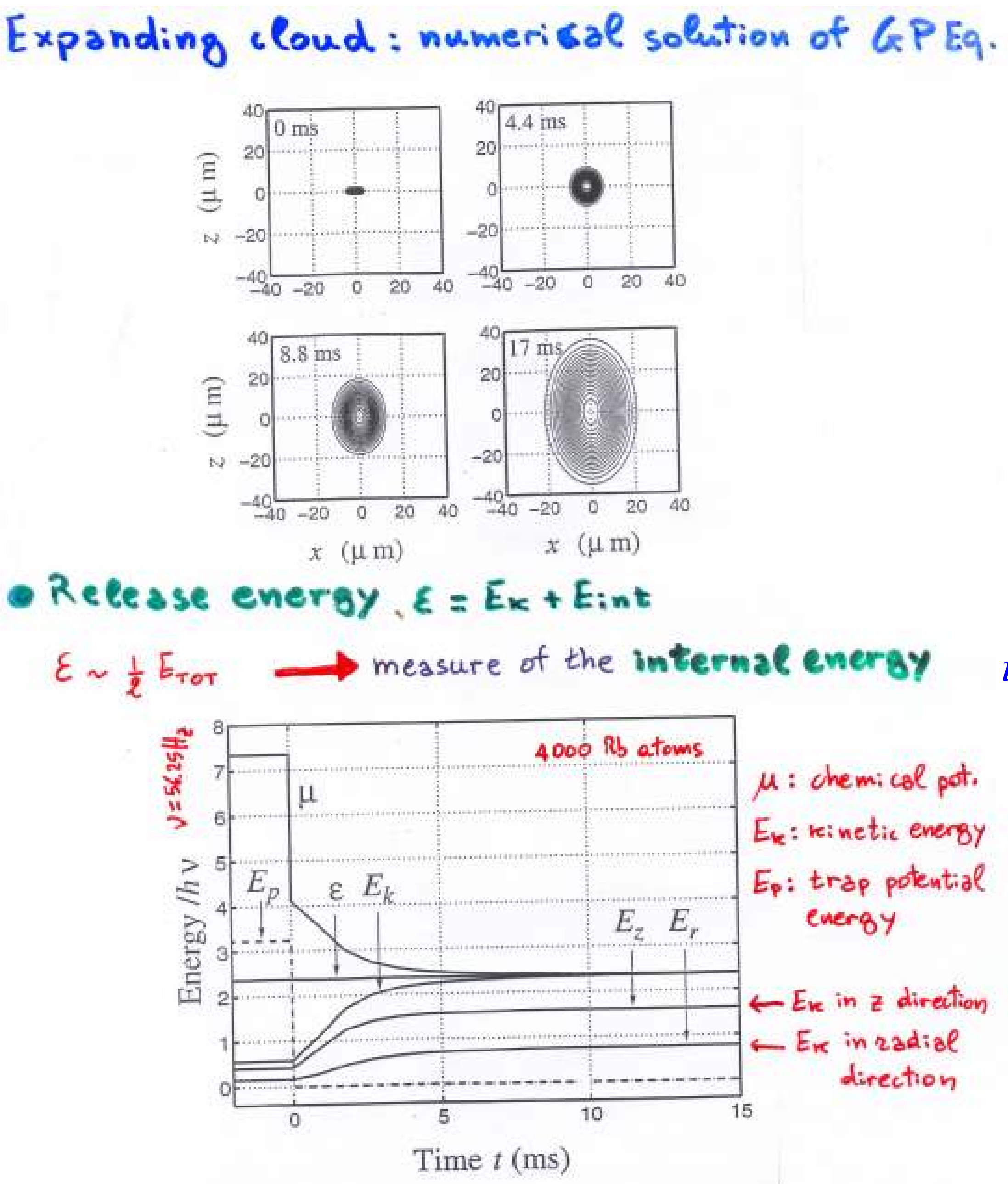
➢ **N: from the 0-th moment of the density distribution**

➢ **Kinetic energy: from $<r^2>$ in ballistic expansion because**

$$<r^2><p^2>\geq|<rgp>|^2=\frac{1}{4}|<\{r,p\}+[r,p]>|^2$$

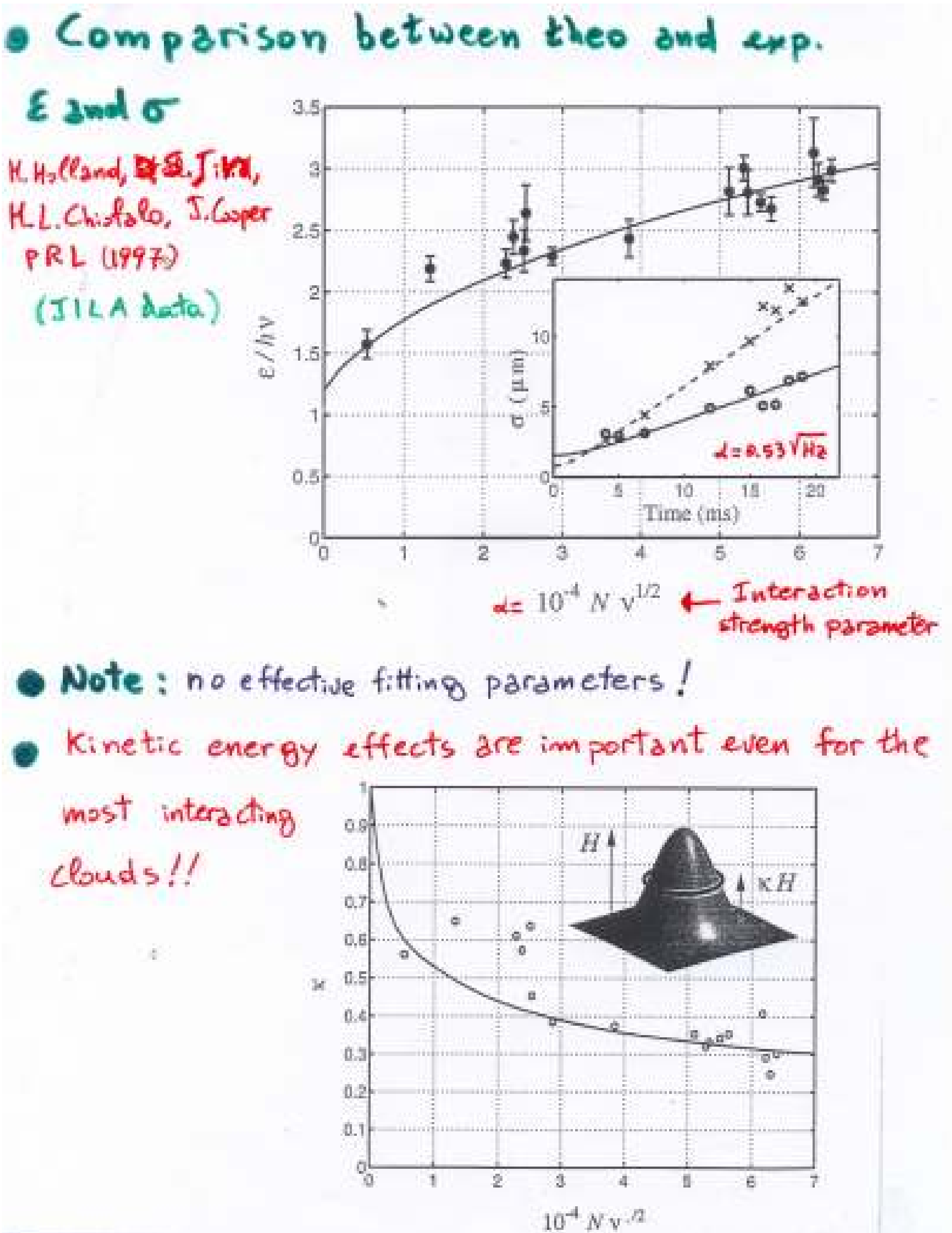
$$\frac{1}{2}m\left(\frac{\partial<r^2>}{\partial t}\right)^2=\frac{1}{8m<r^2>}<rgp+pgr>^2=\frac{<p^2>}{2m}=E_k$$

Past research activity…



Expanding cloud: numerical solution of the GPE equation for the condensate wf. ✡(r,t)

$$i\hbar \frac{\partial \Psi(r,t)}{\partial t} = [-\frac{\hbar^2 \nabla_r^2}{2m} + V_{trap}(r) + \frac{4\pi\hbar^2 aN}{m} |\Psi(r,t)|^2]\Psi(r,t)$$



Past research activity…

➢**This method has been used and is currently in use at JILA to analyse the experimental data**

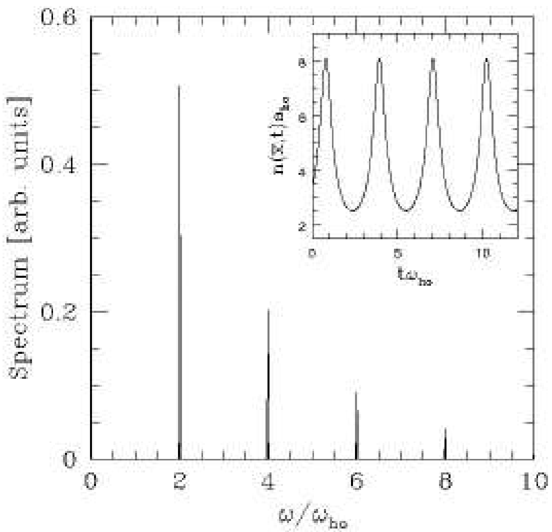
- **The ground-state of 1D hard-core Bosons is in one-to-one correspondence with 1D spin-polarized Fermions in hydrodynamic regime (Tonks-Girardeau)**

  - **The equation for hard-core 1D bosons is similar to GPE but with a 5-th power term**

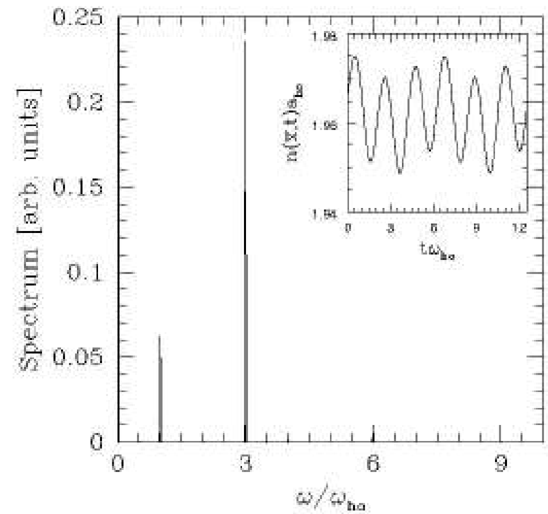
$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = [-\frac{\hbar^2 \nabla_x^2}{2m} + V_{trap}(x,t) + \frac{(\pi \hbar)^2}{2m} |\Psi(x,t)|^4]\Psi(x,t)$$

  - **We have shown that the ground-state and excitation spectrum of 1D harmonically trapped spin-polarized Fermions in the hydro regime can be determined from the numerical solution of this equation as compared to analytical solutions for the Fermi gas in the Thomas-Fermi regime**

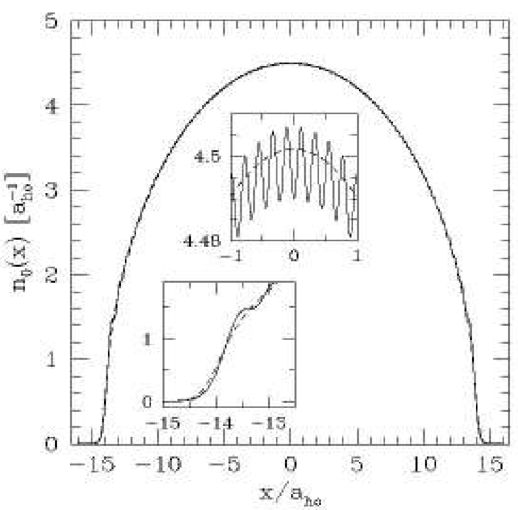
ırch activity…



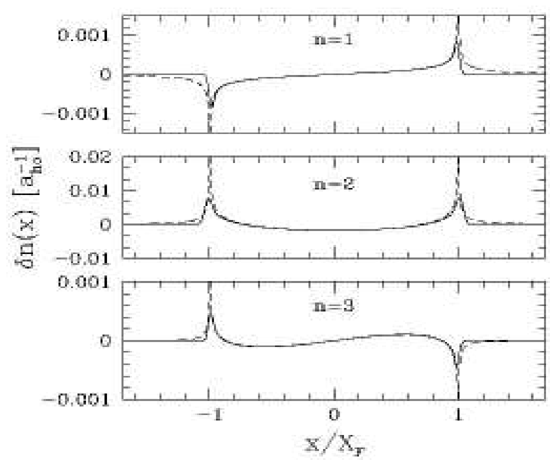
Excitations $\omega=2$◆$_{ho}$



Excitations $\omega=3$◆$_{ho}$



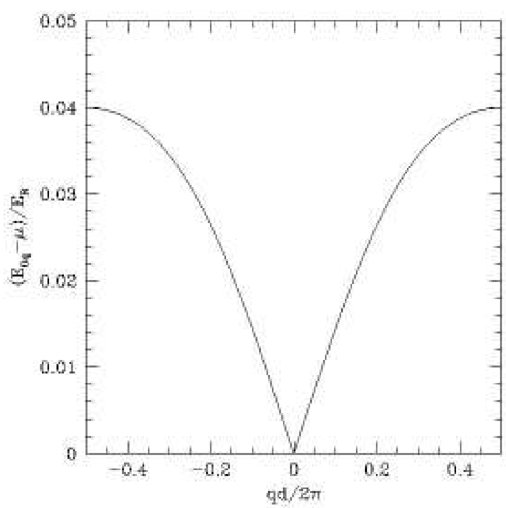
Ground state



Density fluctuations

- **BECs can be trapped in optical lattices realized by standing waves of detuned laser beams with wavelength ●, bringing solid-state physics, quantum atom optics and quantum computing applications at hand in a clean and highly controllable system**

- **First question: how are the BEC energy bands characterized?**



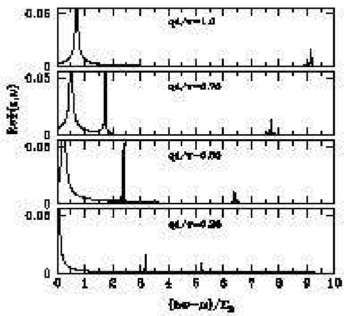
From analysis with in the Wannier representation in the 1D laser potential

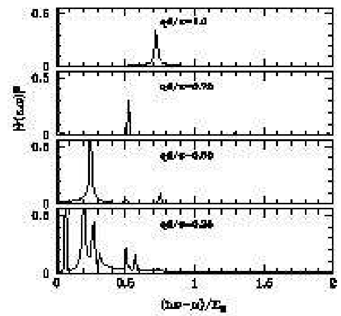
$$V(x) = \alpha E_R \sin^2(2\pi x/\lambda)$$

lowest energy band has linear dispersion at the BZ center as it corresponds to a phase-modulation of the order parameter

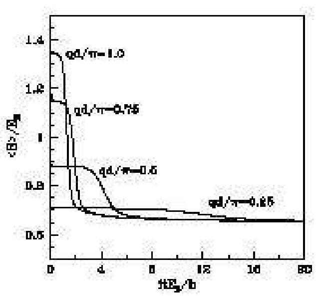
Past research activity…

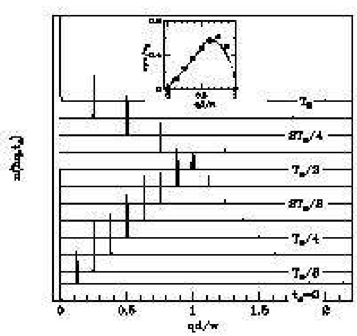
- **How**: using different types of time-dependent external perturbations in the GPE





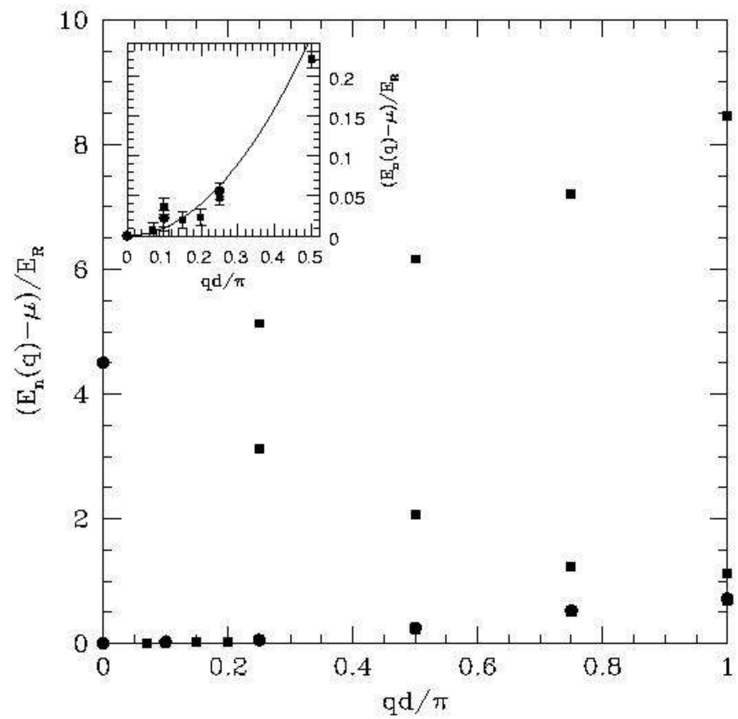
Kicking→velocity spectrum   **Shaking**→density spectrum





Static deformation   **Driving force**

➢ **Second question: how to probe these bands?**

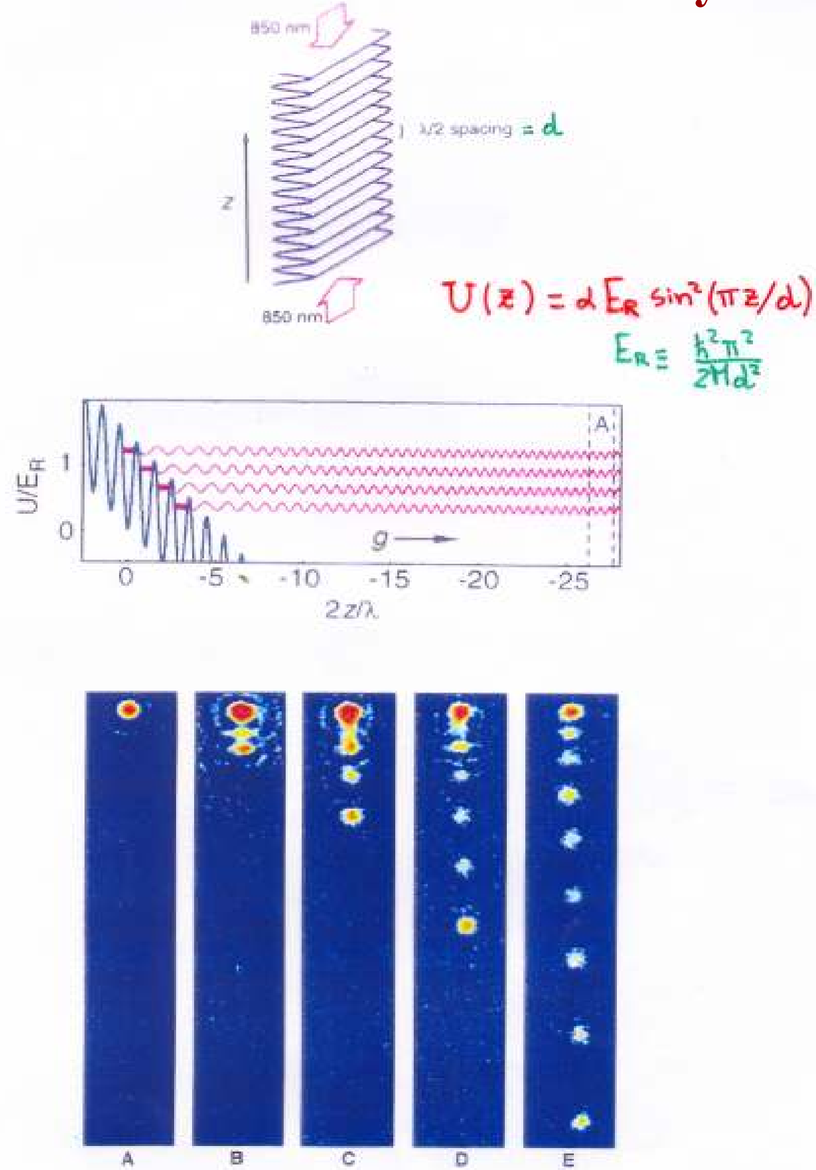


Here nonlinear dispersion as the interactions are not strong enough to make the healing length ☒ < d=●/2

Past research activity…

➢ Application: atom-lasing after coherent emission of matter-wave pulses under the force of gravity

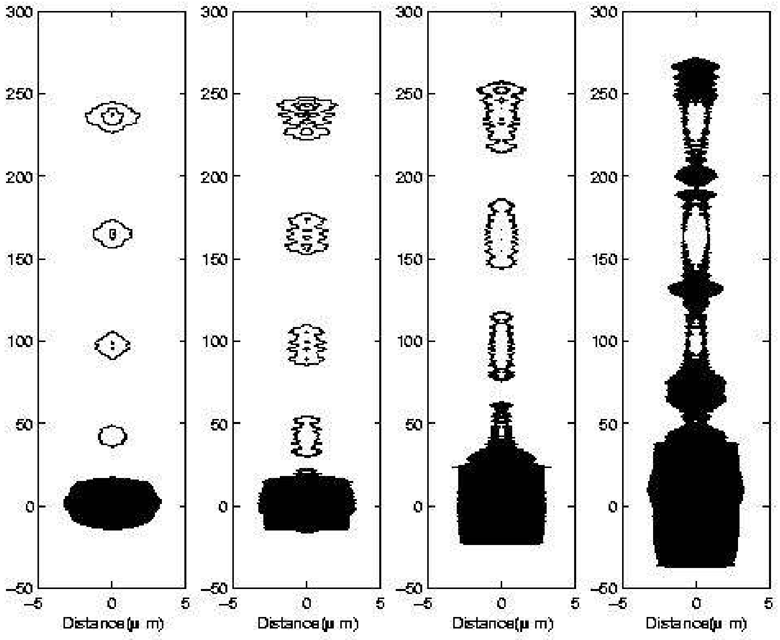
Laser cavity	Brillouin zone
Modulation period T in Q-switching,	Period of Bloch oscillations, T_B
Number of modes in the cavity, n	Number of occupied wells in the lattice
Modes	Condensates at each site
Pulse duration $\Delta T = T/n$	"Width in time" of the parent condensate
Q-switching	Resonant tunnelling to the continuum (also, output coupling)



Anderson & Kasevich, Science (1998)

Past research activity…

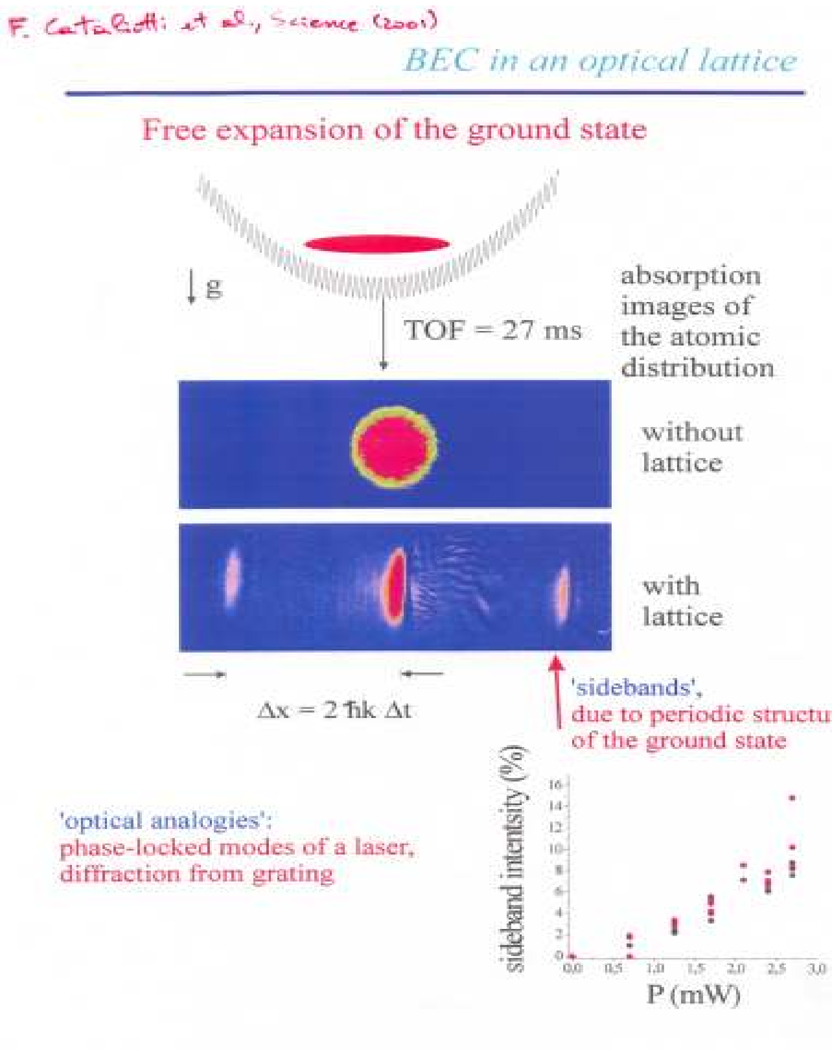
a=0 N=10^5 N=2 10^5 N=3 10^5

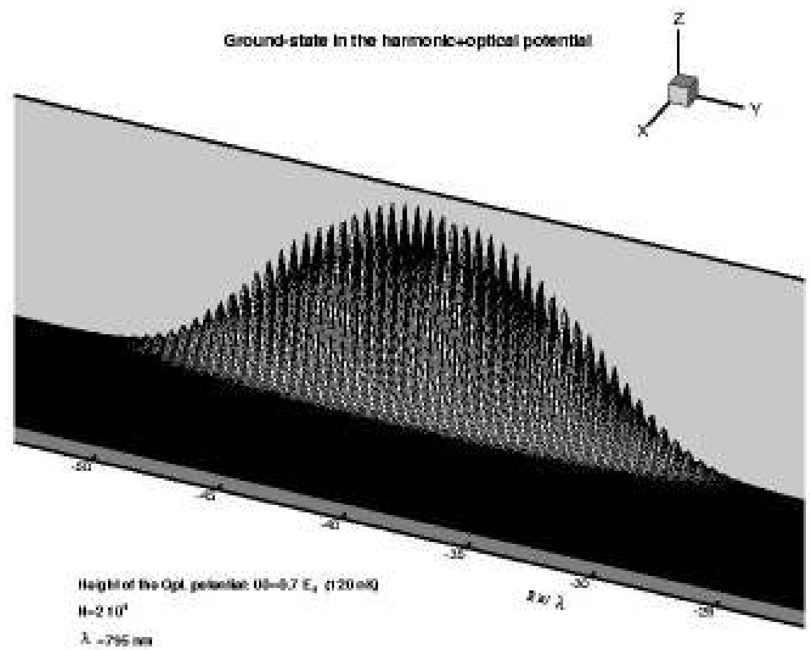


3D simulation of the experiment, after solving the time-dependent GPE by a suitably developed time-marching algorithm

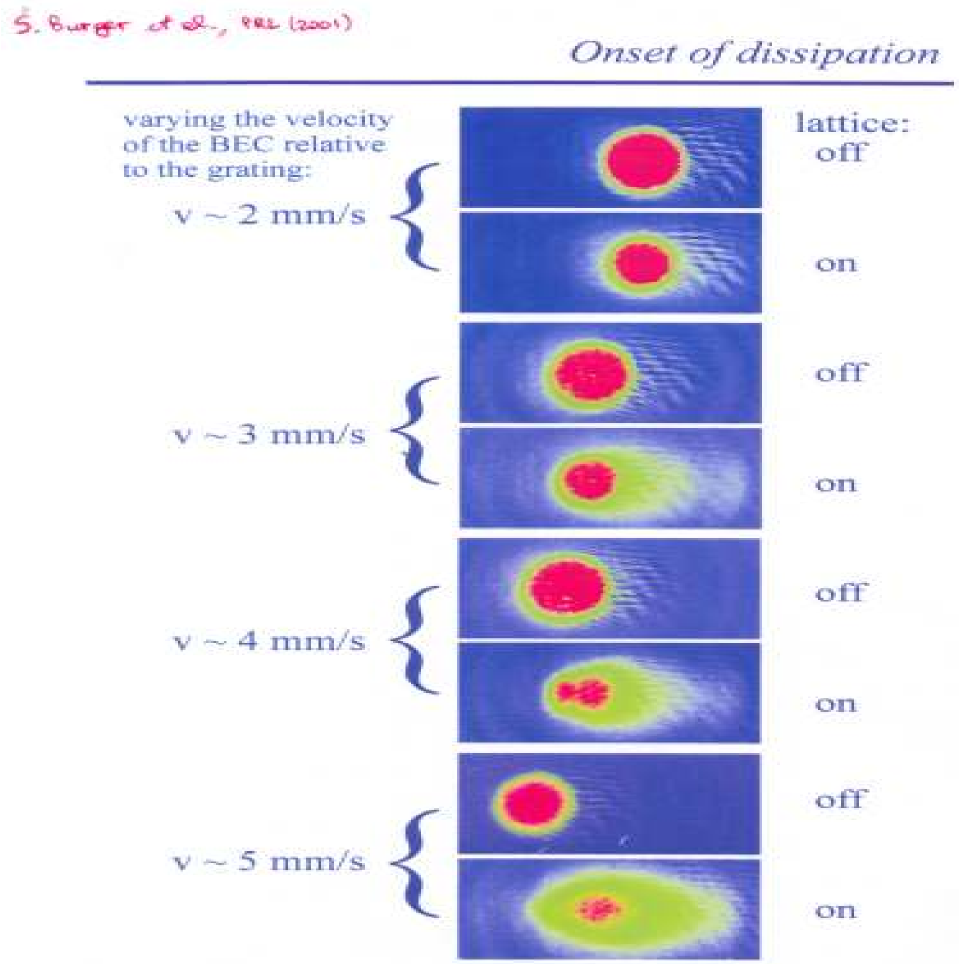
T_{Bloch}=h/mgd=1.1 ms
as in the experiment

➢ **Next question: under which conditions is BEC dynamics in the optical lattice coherent?**

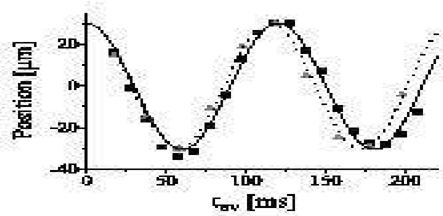




➢ **Experiment: set a BEC to move in an optical lattice+harmonic trap with variable amplitudes of oscillation, barrier height, and interactions (varying the number of atoms)**



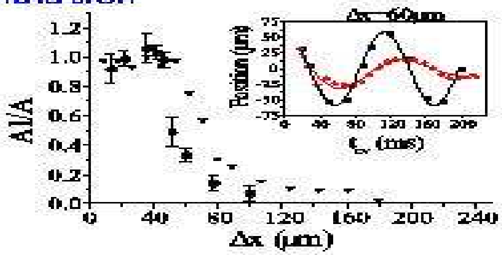
Past research activity…



- Superfluid regime ($\alpha E_R/k_B \simeq 270$ nK, $\Delta x \simeq 31$ μm, $N = 3 \cdot 10^5$): undamped oscillations with a shifted frequency due to the effective mass ($m^*/m = 1.2$)

► **Symbols**: experiment with (■) and without (▲) lattice

► **Lines**: GPE simulation

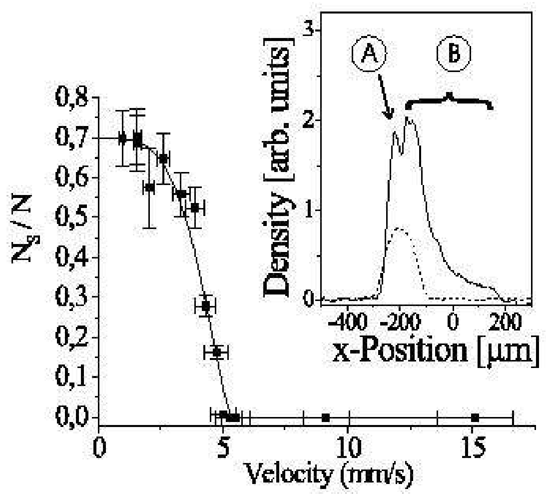


- Dissipative regime: first-peak amplitude of the oscillation decreases with increasing Δx

► **Circles**: experimental data ► **Triangles**: simulation

► **Inset**: Full oscillation for $\Delta x = 60$ μm with ($\star$) and without (■) optical lattice

- Dissipation of superfluidity



❖ **Larger displacements involve larger max velocities**

❖ **Possible superfluid excitations: vortices (not here) and emission of sound waves**

❖ **Density-dep local v_c within TD-DFT**

$$c_s(x) = [(n(x)/M)(\delta\mu/\delta n)]^{1/2}$$

Evaluate $\delta\mu/\delta n$ stretching the 1D BEC

$$c_s^{max} \;;\quad 5.2mm/s[\exp:(5.3 \pm 0.5)mm/s]$$

$$c_s^{min} \;;\quad 3mm/s[\exp:(3 \pm 0.5)mm/s]$$

❖ **Solid line is a fit assuming a density-dependent critical velocity and a parabolic envelope of the density distribution (as it is the case under exp. conditions)**

➢ The BEC dynamics in the optical lattice under constant + harmonic forces can be also viewed as Josephson-type effects

❖ **Inserting** $q(t) = Ft/\text{h} + m\omega A\sin(\omega t + \phi_0)/\text{h}$ **in the semiclassical expression for** $\overline{p}(t)$ **and expanding in Bessel functions**

$$\overline{p}(t) = \frac{\text{h}}{d}\frac{\sum_{l=1}^{\infty}[-2\partial g_0(ld)/\partial l]\sum_n(-1)^n J_n(lAd/a_{ho}^2)\sin[2\pi(Fd/h - n\nu)t - n\phi_0]}{1 + 2\sum_{l=1}^{\infty} g_0(ld)\sum_n(-1)^n J_n(lAd/a_{ho}^2)\cos[2\pi(Fd/h - n\nu)t - n\phi_0]}$$

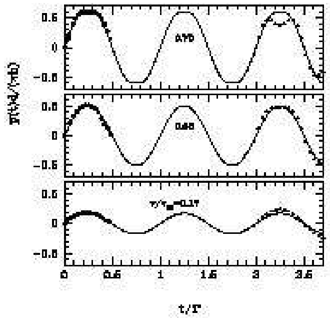
$$a_{ho} \equiv \sqrt{\text{h}/m\omega};\ |\text{F}|\text{d}/h \equiv \nu_{Bloch};\ \nu = \omega/2\pi$$

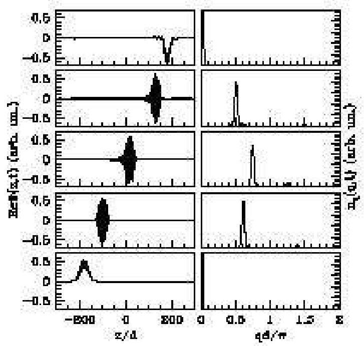
For l=1: $\overline{p}(t) = p_0\sum_n(-1)^n J_n(Ad/a_{ho}^2)\sin[2\pi(\nu_{Bloch} - n\omega)t - n\phi_0]$

❖ **Formally equivalent to a Josephson weaklink current**
IJ=Icsin☎👎♐(t)➂ with 2eV=Fd, 2eU=m◆🗎Ad, and
👎♐(t)🖬q(t)d

- **Applications: harmonic force**

Multimode oscillations



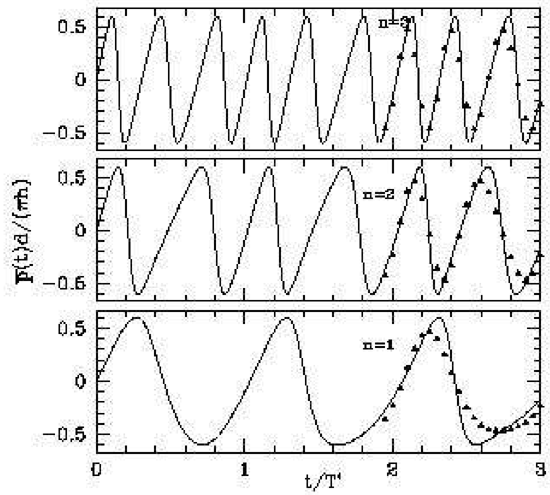


- May be useful for:
  - Tailoring matter-wave pulses
- Observability: TOF measure of the momentum distribution

Past research activity…

- **Applications: constant ⊕ harmonic forces**

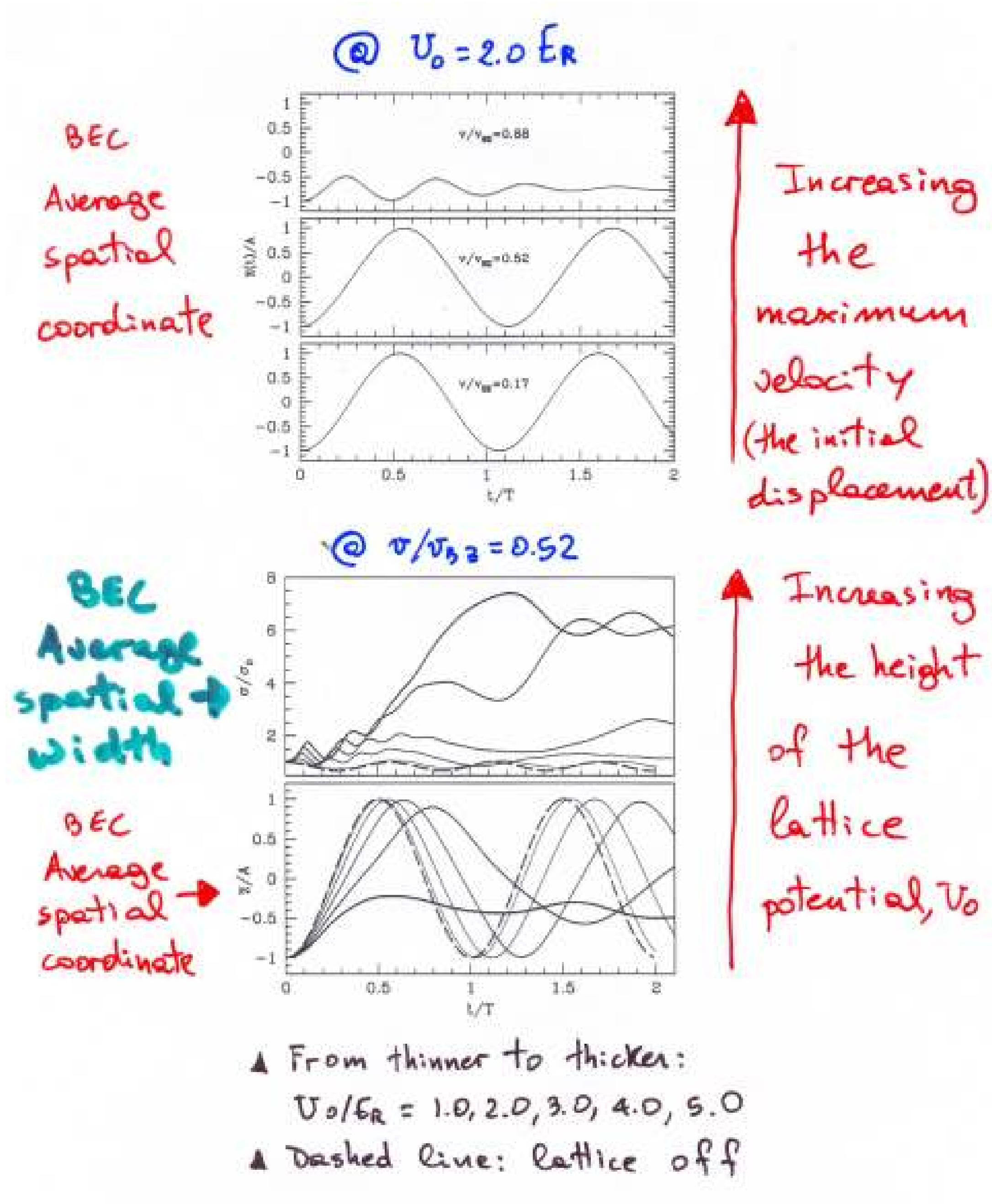
Multiple resonances



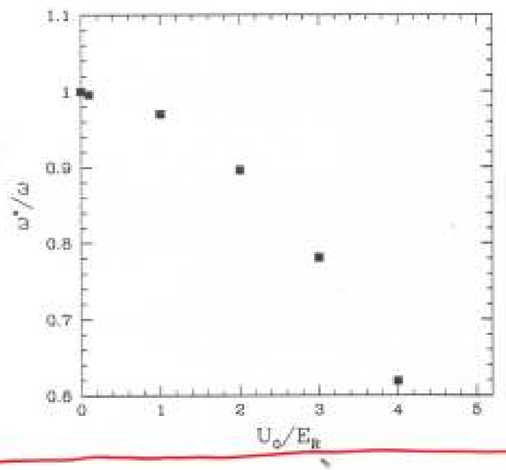
- May be useful for:
  - Precision measurements of frequencies **and forces**
  - Precision measurements of anharmonicity
- Observability: TOF measure of the momentum distribution

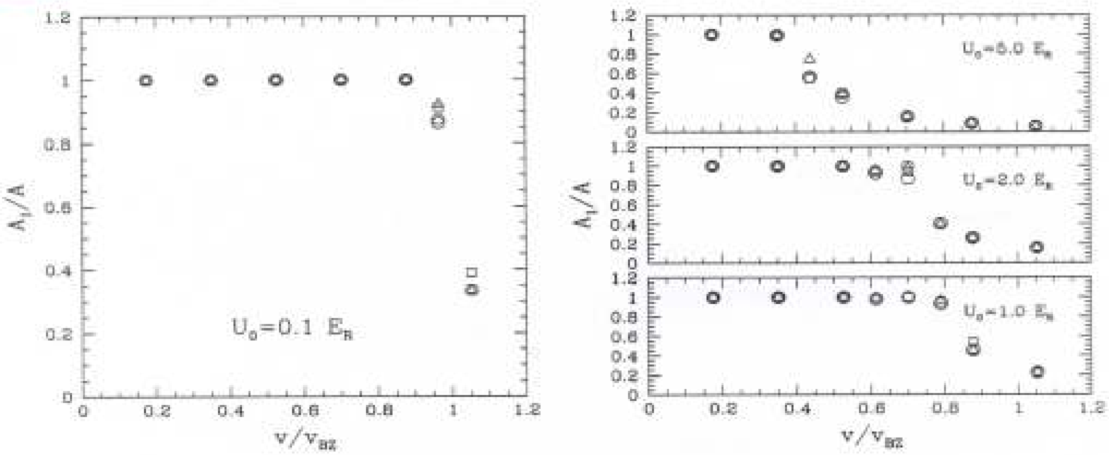
Past research activity…

❖ **From coherence to decoherence**



▲ Frequency ω* of c.m. oscillation





▲ Scaled amplitude A1/A of the first peak in the c.m. oscillation: phase diagram v - U0 - N

△ N = 10⁴
□ 5·10⁴
⬡ 10⁵
○ 3·10⁵

Past research activity…

❖ **Results consistent with those obtained by more sophisticated methods. We have indeed solved the ultracold atom dynamics with an on-purpose developed Monte Carlo Particle-In-Cell method, suited also for fermionic atoms**

➢ With these nonlinearities (external potential, interactions) at hand, is there the possibility of realizing chaotic behavior?

❖ Classical chaos: high-sensitivity to initial conditions, with variables eventually filling up the entire phase space. Tutorial example: δ-kicked rotor

$$H = \frac{p^2}{2} - \frac{K}{T}\cos(x)\sum_m \delta(t - mT)$$

Mapping: $p_{n+1} = p_n + K\sin(x); x_{n+1} = x_n + p_{n+1}$

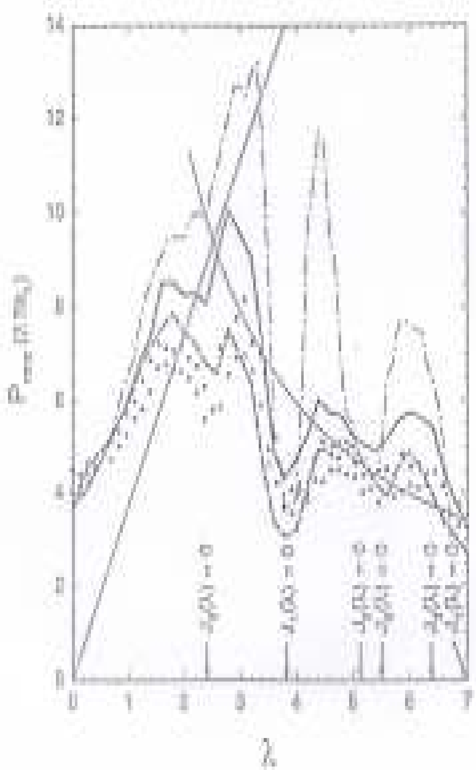
When resonances overlap $(2/\pi)K^{1/2} > 1$ $\langle \Delta p \rangle$ diffuses away

Only one parameter K drives the transition

❖ Quantal : signatures are different, as dynamial localization may occur due to destructive intereference of discrete levels. Also, diffusion doesn’t last forever, since phase-space is binned in units of h. Two parameters K and h drive the transition

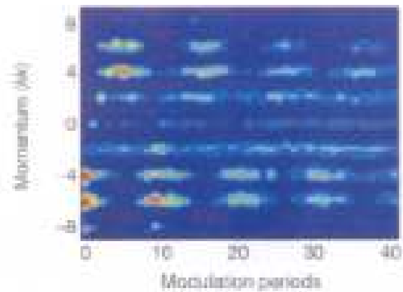
Past research activity…

[Moore, Robinson, Bharucha, Williams & Raizen,
e.g. PRL 73, 2974 (1994)]



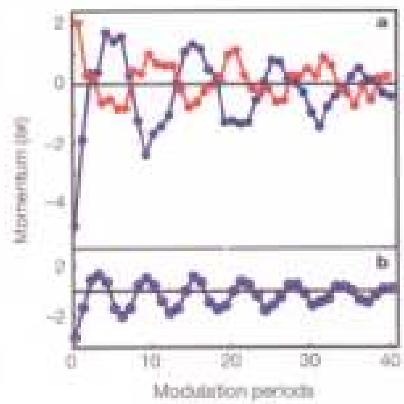
^{23}Na ultracol
atoms

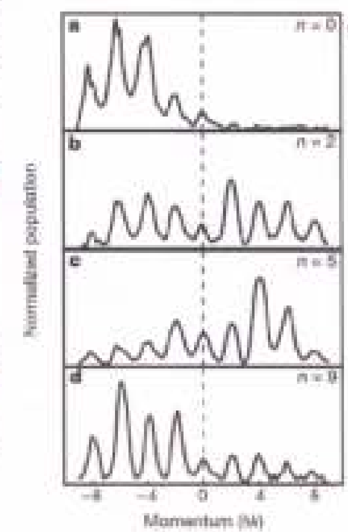
• Chaos in BECs, Hensinger et al., Nature 2001



$$H(t) = \frac{p^2}{2} + 2K\left(1 + 2\frac{A}{\alpha}\sin\Omega t\right)\sin^2(x/2)$$

At t=0, BEC is put in a Floquet state trapped into resonance p_0





The BEC momentum distribution oscillates between Floquet states with $\pm p_0$

Past research activity…

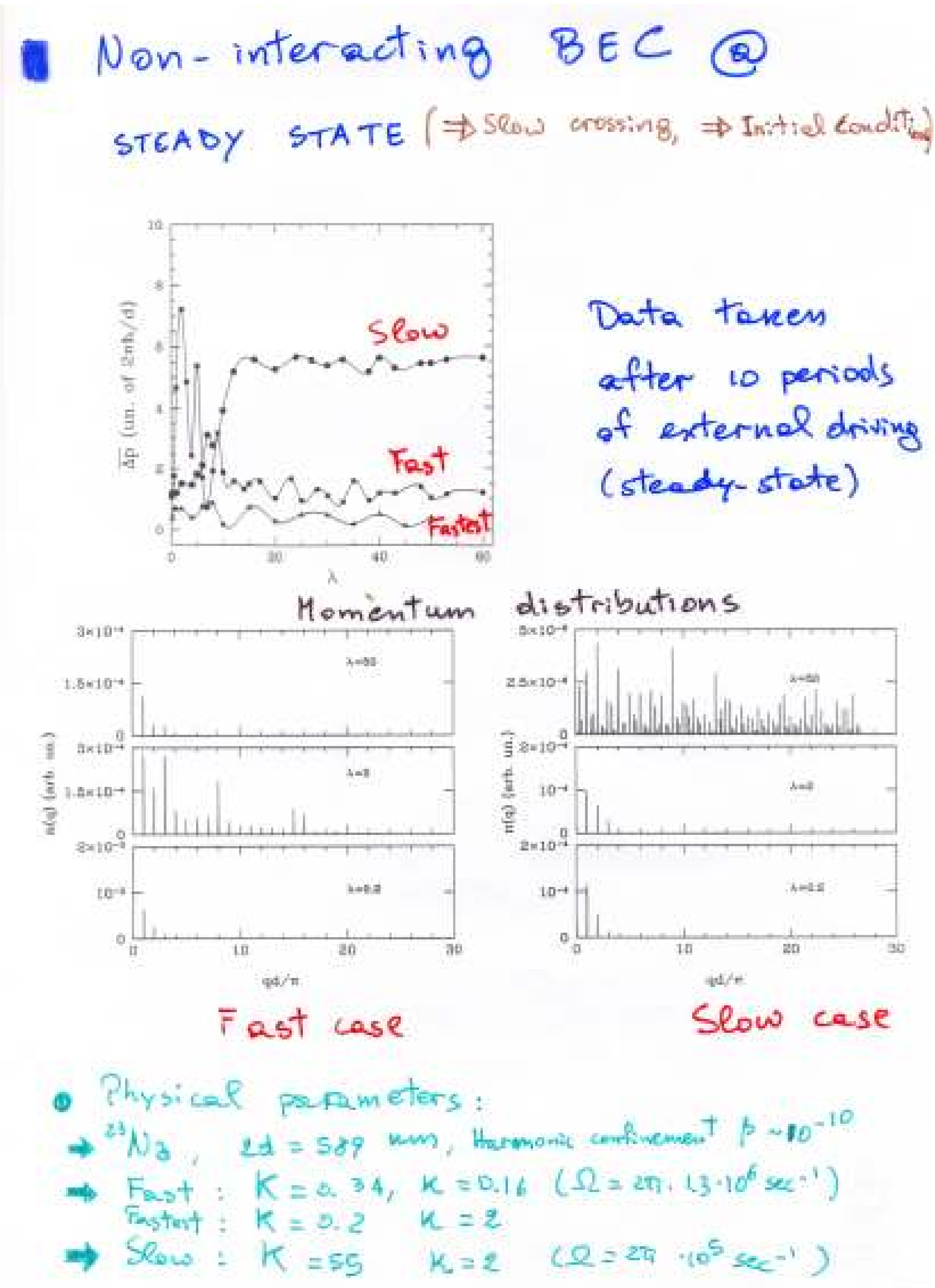
❖ Generalization of kicked rotor in adimensional form:

$$i\kappa\dot{\Phi} = \left(-\frac{\kappa^2\nabla^2}{2} + K\cos(x-\lambda\sin(t)) + g\,|\Phi(x,t)|^2\right)\Phi(x,t)$$

Fast crossing K<<λ (recovers kicked rotor)
Slow crossing K>>λ (mostly unknown)

❖ Questions:

Role of initial conditions and interactions (BEC, cold atoms) on
-Quantum breaking time for transition from classical to quantum chaos (e.g. when average kinetic E saturates) different for slow (0.22 x period T) and fast crossing (2.8 T)
- Decay of phase correlations: exponential or polynomial?



Past research activity…

Found that:

❖**Fast case corresponds to Raizen experiment**

❖ **Interactions do not significantly affect the quantum breaking time**

❖ **Interactions are responsible for exponentially decay of phase correlations in both fast (more evident) and slow (less evident) crossing**

Current and Future…

Resonant Superfluidity in Quantum Degenerate Fermi gases

- ❑ Motivations: Exp&Theo context
- ❑ The original predictionof RS
- ❑ Open theoretical problems: generalaties
- ❑ Open theo problems: dynamical effects
- ❑ Open problems: universality

[PRL01,PRL02,PRA02,PRA04,PLA04,PRL sub]

-Project granted by SNS (coordinator)
-Proposal including also novel quantum states in dipolar gases and with disorder submitted to CNISM (coordinator)
Collaborators: M. Holland'group@JILA, S.Kokkelmans@Eindhoven, D. Jin's group@JILA, S. De Palo@Trieste, R.Citro@Salerno, M. Marinaro@Salerno, S. Giorgini@Trento, C. Menotti@Trento, K.Levin's group@Chicago

❑Motivations I: Fermion pairing is an evergreen story

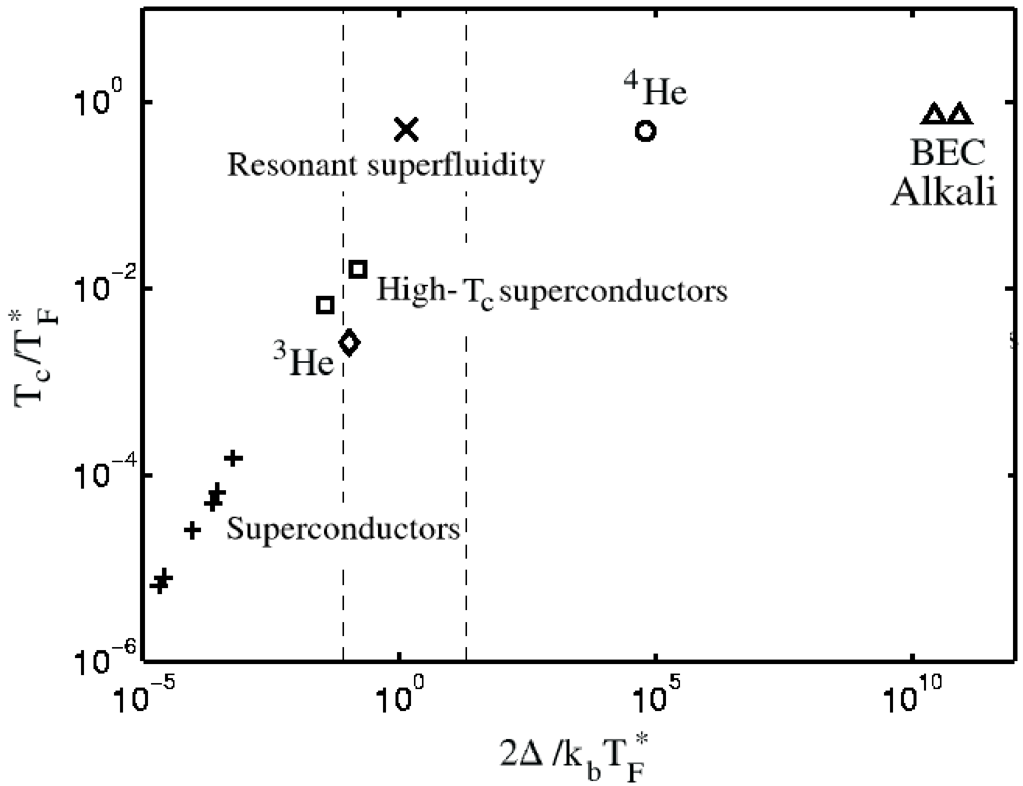
Fermion pairing is key-concept in understanding non-trivial effects in c-mat

⇨ **Superfluidity (superconductivity) is intimately connected to BEC**

Strength and type of Interactions, and Dimensionality determine

⬇

Nature and Symmetry of Normal and Super State, T_c and Δ



[Holland, Kokkelmans, MLC&Walser, 01]

Bosonic fluids

• ^{4}He [Sokol 93] : strongly interacting so that n_c<10% while n_s= 100% at $T \to 0$

• Alkali BEC [JILA and MIT 1995] very special indeed: so cold to afford enough diluteness and n_c , n_s=100% at $T \to 0$

Fermionic fluids

• ^{3}He [Leggett 1975] : p-wave for interaction is SR repulsive+LR weakly attractive

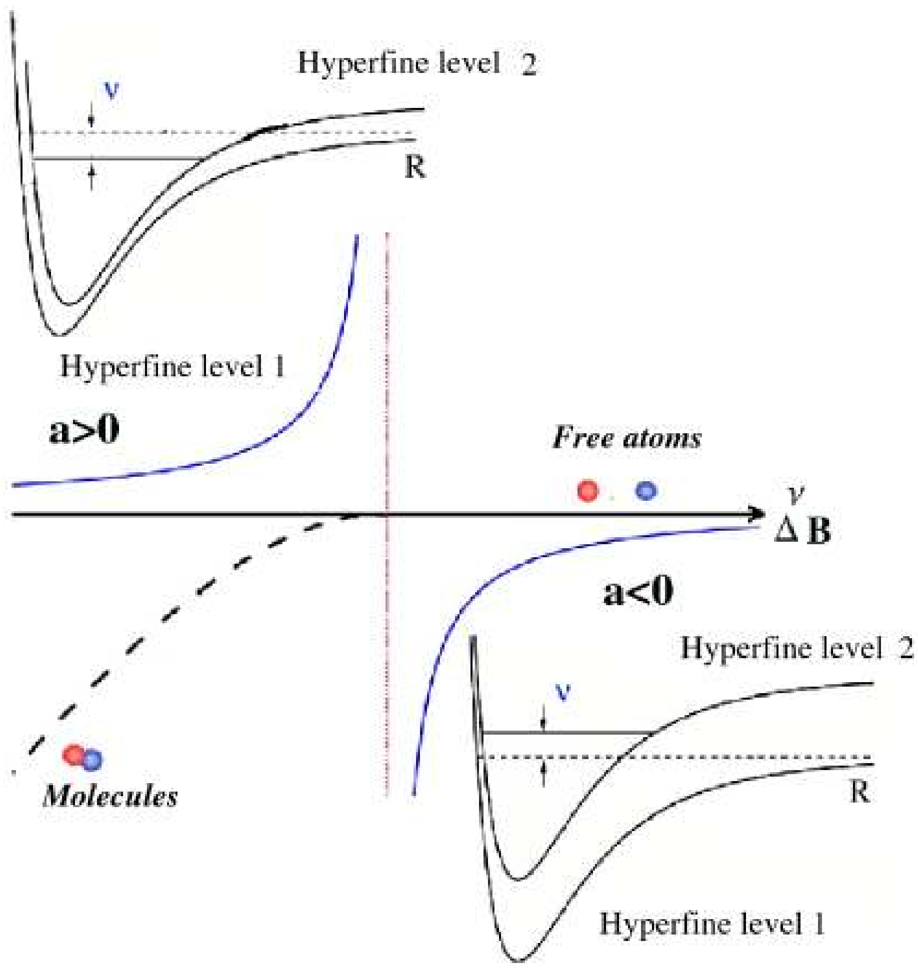
• HTSC [Bednorz&Muller 1987] : strong SR correlations acting on charge and spin compete to make $T_c / T_F : \ 10^{-2}, k_F\xi : \ 5$

….and even more examples of fermion pairing….

- **Dimensionality:** **Integer and Fractional Quantum Hall Effects in degenerate 2D electron gas under strong magnetic fields – Composite Fermion Theory** [Jain 1989]

- **Type of interactions:** **BEC of excitons in semiconductor structures – non conclusive observations** [Mysyrowicz *et al.* '90s, Butov *et al.* 2002]

❑Motivations II: experiments in (Bose and Fermi) atomic gases do control Temperature Interactions Dimensionality



Control of Interactions:

Fano-Feshbach resonances

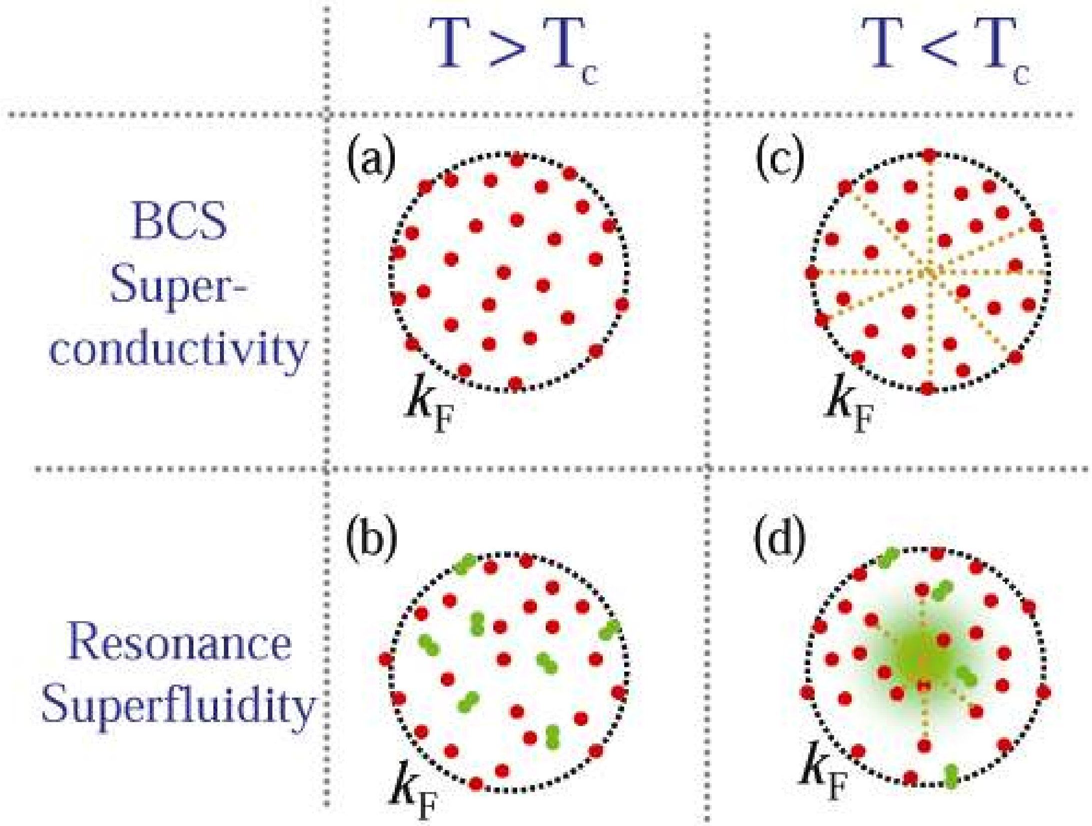
"Control" of Temperature:

Sympathetic/Evaporative cooling

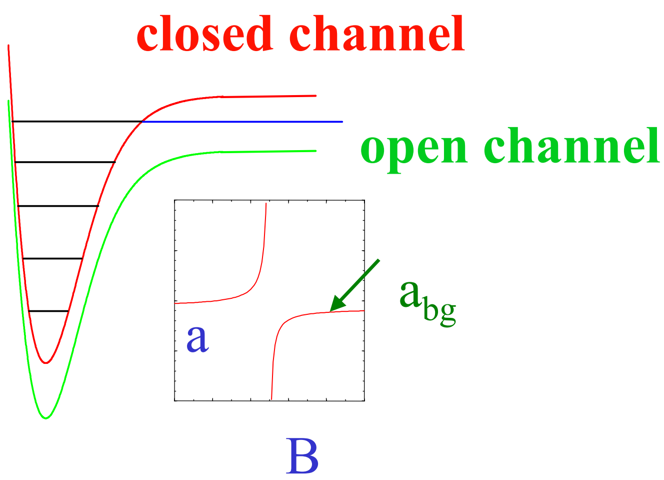
Control of Dimensionality:

Possible but not yet exploited in Fermi gases

❑ Resonance superfluidity: the original idea

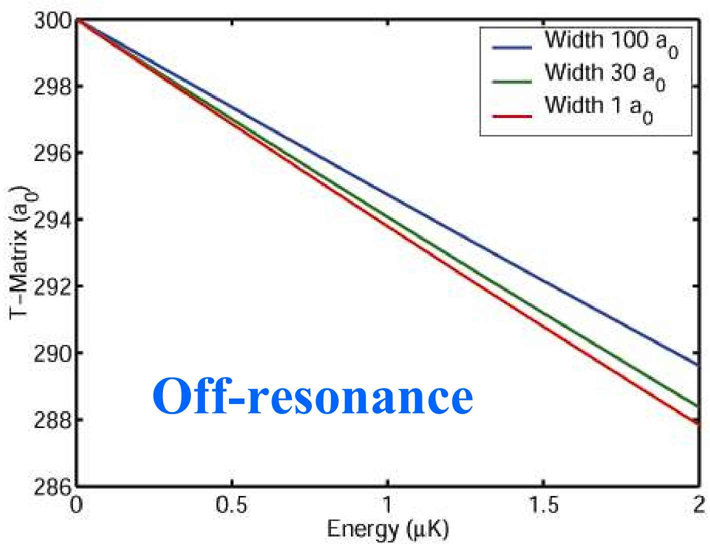


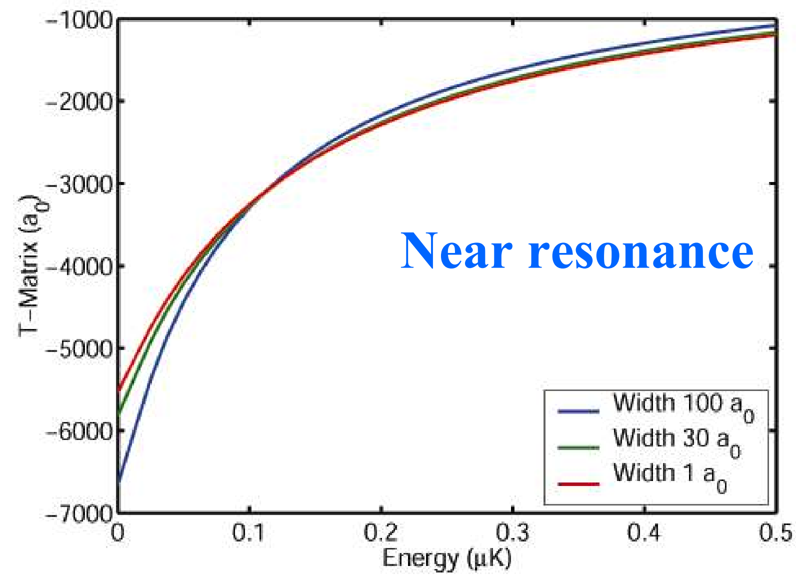
Two different spin states $|\downarrow\rangle$ and $|\uparrow\rangle$ ➡ Interaction Hamiltonian [PRL 2001]:



- **Short-range molecular state**
- **Relatively long-lived molecules**
- **Scattering becomes energy-dependent**
- **Recovering BCS-like pairing at $\nu \to \infty$**

$$H = \sum_k \varepsilon_k (a^+_{k\uparrow} a_{k\uparrow} + a^+_{k\downarrow} a_{k\downarrow}) + \sum_k \nu b^+_k b_k$$
$$+ U_{bg} \sum_{k_1 \ldots k_3} a^+_{k1\uparrow} a^+_{k2\downarrow} a_{k3\downarrow} a_{k4\uparrow}$$
$$+ g \sum_{kq} b^+_q a_{q/2+k\uparrow} a_{q/2-k\downarrow} + b_q a^+_{q/2-k\downarrow} a^+_{q/2+k\uparrow}$$



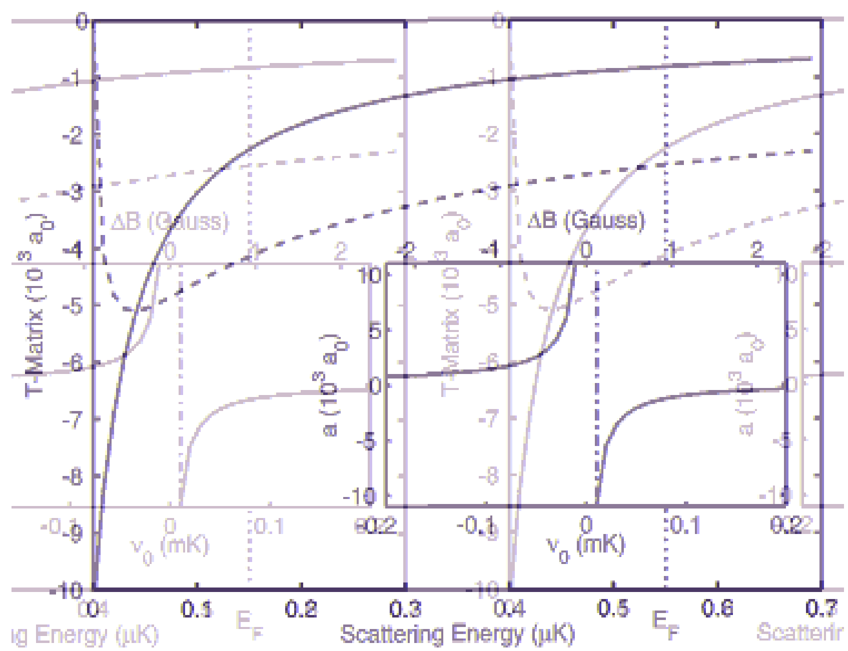


Renormalization cut-off K is needed

$$U_{bg} \rightarrow \overline{\mathrm{U}}$$

$$g \rightarrow \overline{\mathrm{g}}$$

$$\nu \rightarrow \overline{\nu}$$



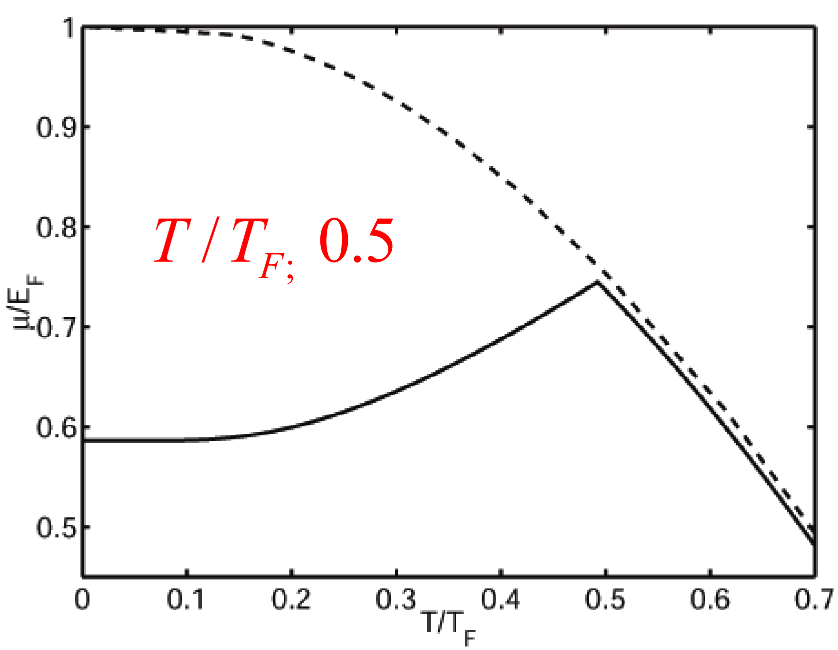
[PRA 2002]

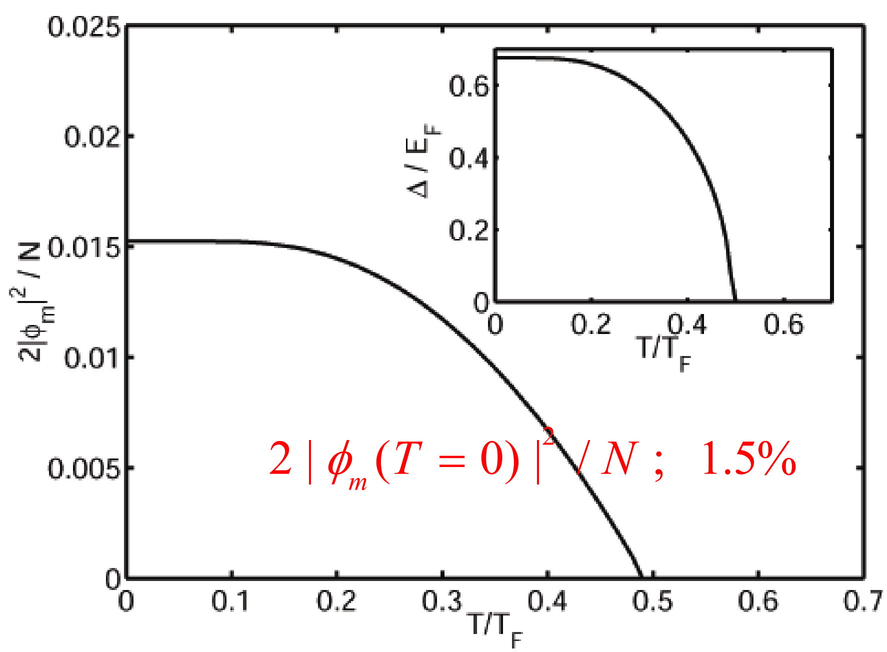
Self-consistent Hartree-Fock-Bogoliubov solution obtained diagonalizing H after introduction of the following mean fields:

Molecular field $\phi_m = < b_0 >$

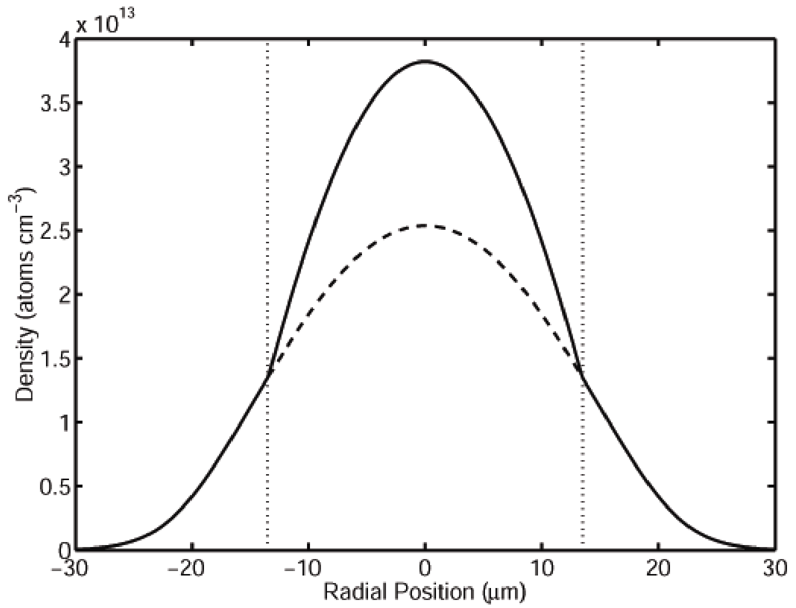
Normal fermionic density $n = \sum_{k\sigma} < a^+_{k\sigma} a_{k\sigma} >$

Pairing field $p = \sum_{k} < a_{k\uparrow} a_{-k\downarrow} >$





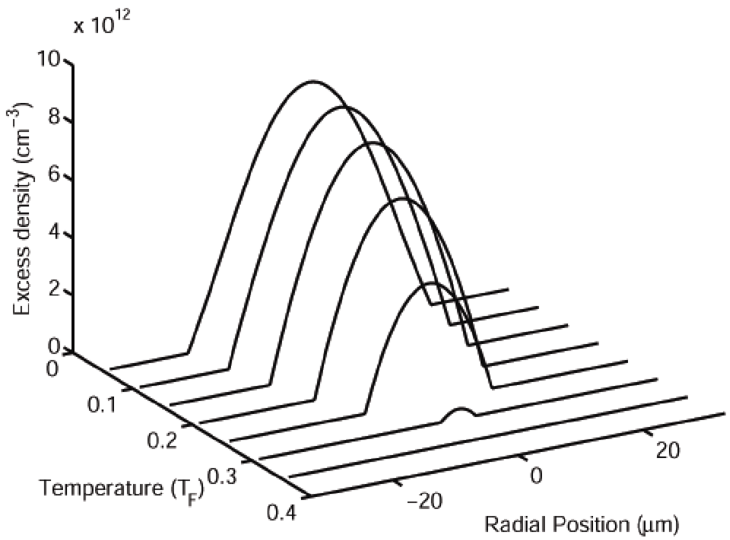
Solution in the trap by means of Local Density Approximation

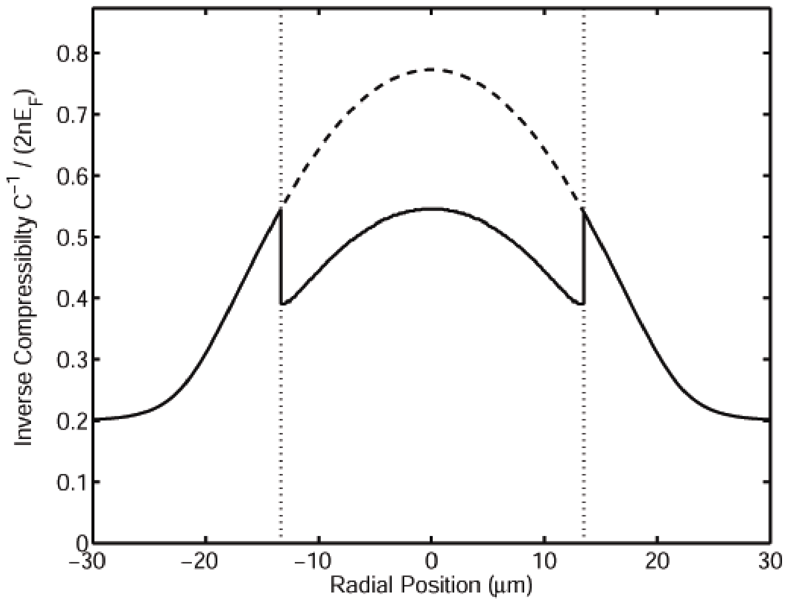


Emergence of superfluidity as a bulge in the trap center

Compressibility changes

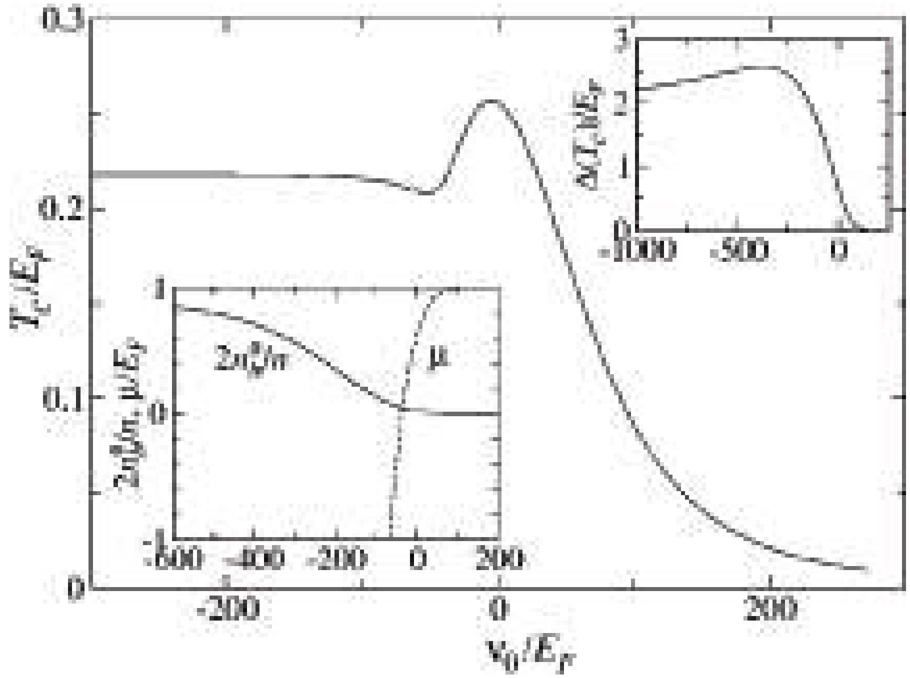
[PRL 2002]

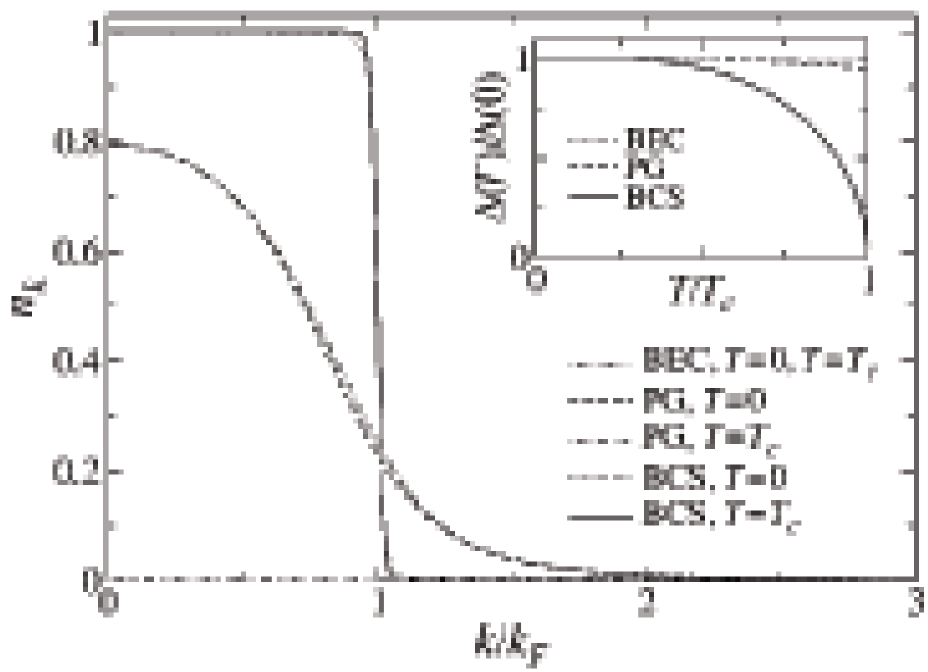




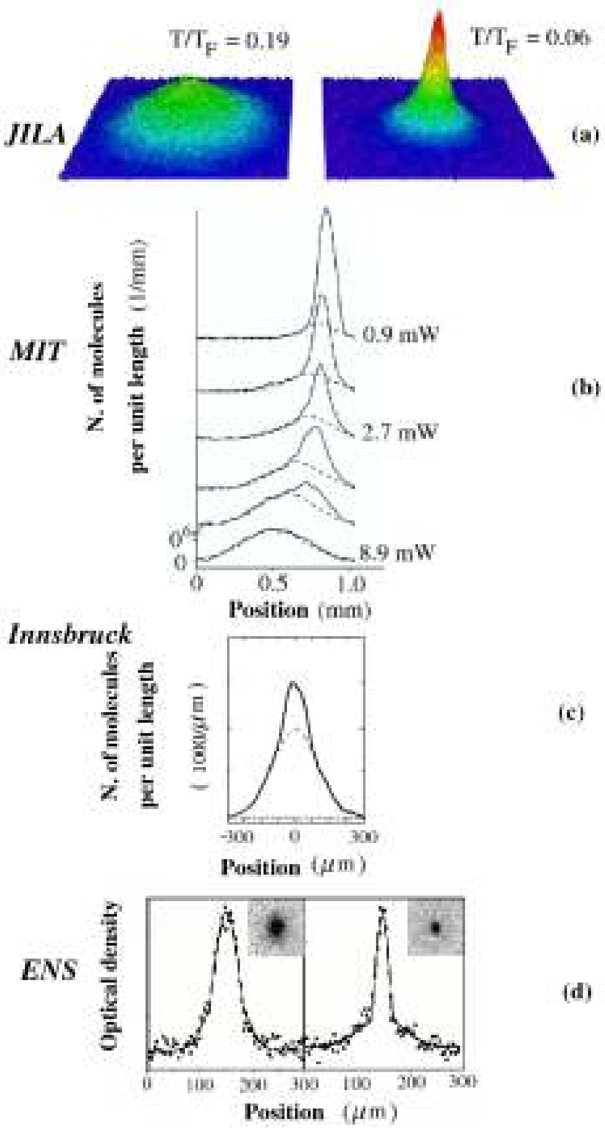
Introducing the non-condensed bosons $m = \sum_k < b_k^+ b_k >$

[PRA 2004]





BEC of "2-fermion molecules"



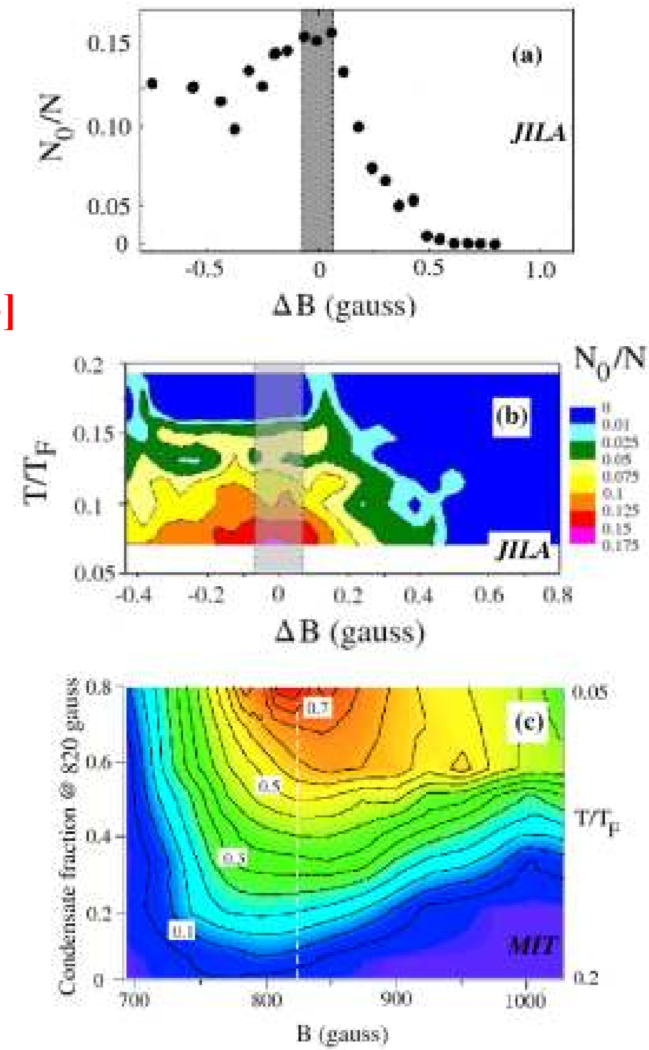
[Greiner *et al.* '03]

[Zwierlein *et al.* '03]

[Jochim *et al.* '03]

[Bourdel *et al.* '03]

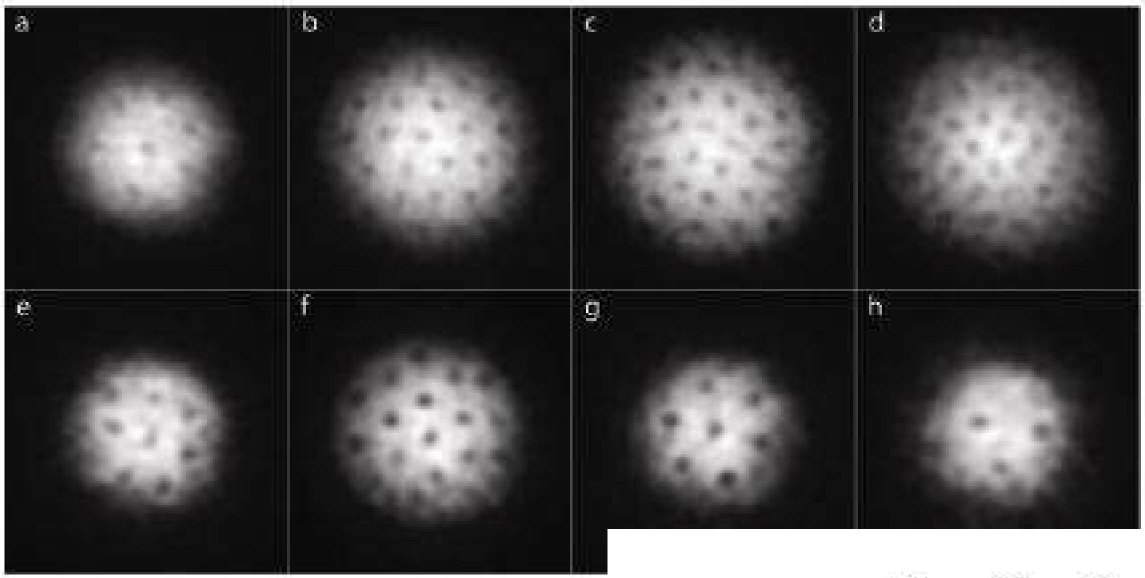
Tuning across resonance

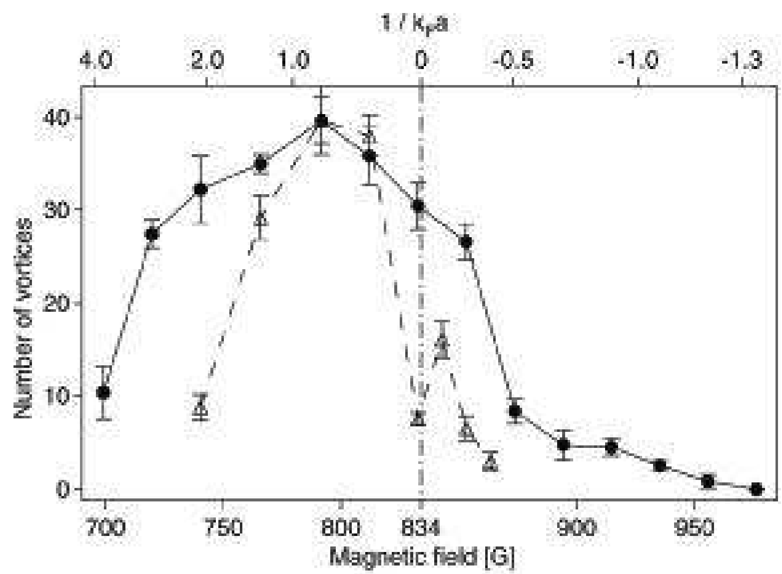


[Regal *et al.* '04]

[Zwierlein *et al.* '04]

Observation of superfluid state





[Zwierlein *et al.* ‘05]

❑ (Open) Theoretical Issues on the Crossover from Bosonic to Fermionic Superfluidity I

The key-point:

- **The formation of Cooper pairs and their condensation to the lowest energy state do not necessarily occur at the same time (BCS in metallic SC is exception!)**

- **Tuning of attractive or resonant interactions may create pairs that populate higher energy states (on the energy scale of the interactions) and leave states $\approx \Delta$ above E_F depleted**

→ **(pseudo)gap formation**

➢ **Pairing field may build up as a propagating ($k \neq 0$) mode because of fluctuations in its**

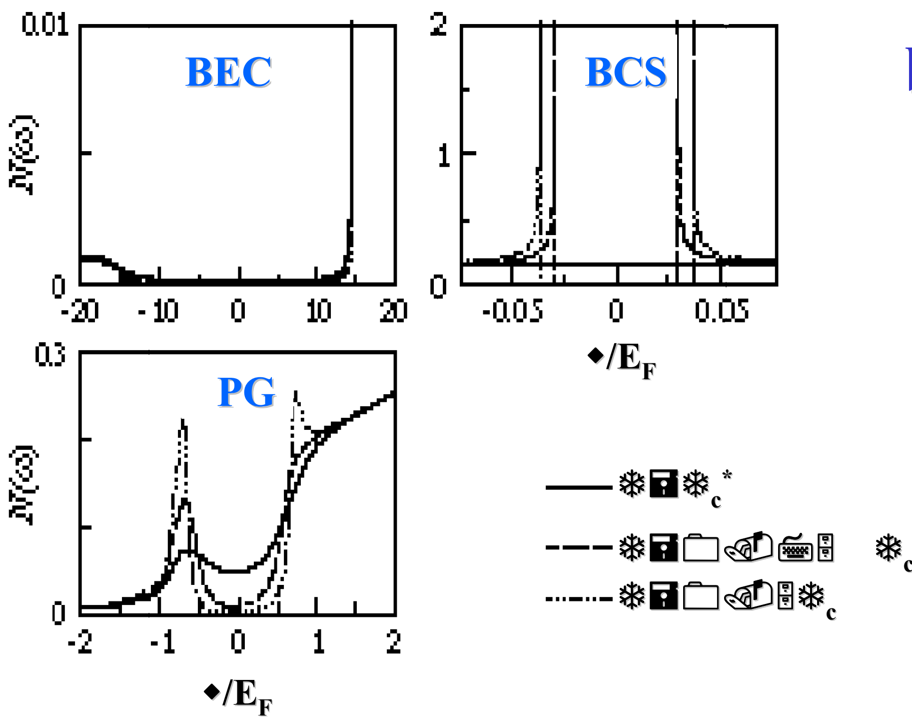
✓amplitude

or

✓phase

[J. Stajic, J. Milstein *et al.*, PRA ’04]

Density of states N☎◆✆



below a temperature $T^* \approx \Delta/k_B$

Complete phase locking occurs only below $T_c < T^*$

Crossover

between two extreme limits depending on pair size ☒

BCS ("fermionic") *n* ☒$^{▤}$ >>*1*

BEC ("bosonic") *n* ☒$^{▤}$ <<*1*

❑ Theoretical Issues on the Crossover… II

1. Observability

➢Normal *vs.* Superfluid State

Signatures of S-state: transverse probes (see MIT experiment!)

Signatures of N-state: (pseudo)gap (Innsbruck, JILA,…)

Collective exc.: only if collisional regime known (Innsbruck, Duke,…)

➢Bose-Einstein Condensation *vs.* Superfluidity

Interactions make n_c differ from ❑$_s$

Phase-coherence probes

➢Dynamics *vs.* Thermodynamics [(non)equilibrium]

Role of dynamical effects in the formation of the pairs (ENS, Innsbruck, Duke, JILA)

❑ Theoretical Issues on the Crossover…II

2. Models

Single-channel (only c-fermion)

with single parameter

$$H = \sum_{k\sigma} \varepsilon_k c^+_{k\sigma} c_{k\sigma} + \sum_{qkk'} V_{kk'} c^+_{q/2+k\uparrow} c^+_{q/2-k\downarrow} c_{q/2-k'\downarrow} c_{q/2+k'\uparrow}$$

$V_{kk'} \leftrightarrow a_F$ at given ν

Two-channel (b-boson/a-fermion)

with three parameters

$$H_{res} = \sum_{k\sigma} \varepsilon_k a^+_{k\sigma} a_{k\sigma} + \sum_q \left(\frac{\varepsilon_q}{2} + \nu\right) b^+_q b_q + \sum_{qk} g_k (b^+_q a_{q/2-k\downarrow} a_{q/2+k\uparrow} + hc) + \sum_{qkk'} U_{kk'} a^+_{q/2+k\uparrow} a^+_{q/2-k\downarrow} a_{q/2-k'\downarrow} a_{q/2+k'\uparrow}$$

$U_{kk'} \leftrightarrow a_{F,bg}$: background a_F

ν : detuning from resonance

g : coupling close-open channel

Single-channel with single parameter *a*

Eagles (1969), Leggett (1980), Nozieres&Schmitt-Rink (1985)
Electron gas, BCS ground-state with large attractive interactions

Randeria *et al*.(1992), Chen *et al*.(1999), Pieri&Strinati (2000)
Electron gas, higher-order expansions from BCS

Perali *et al*.(2003)
Atomic Fermi gases with Fano-Feshbach resonances, higher-order expansions from BCS

Two-channel (boson-fermion) with three parameters

Ranninger&Robaszkiewicz (1985), Friedberg&T.D.Lee (1989), MLC et al. (1995),…
Electron gas, BCS-like ground state

Holland *et al*., Timmermans *et al.* (2001), Chiofalo *et al*. (2002), Ohashi&Griffin (2002), Milstein *et al*. (2002), Stajic&Milstein *et al.* (2004)
Atomic Fermi gases with Fano-Feshbach resonances, BCS-like with effective interaction mediated by pairs (the "phonons")

➢ **Share: key-concept of non-simultaneous pair formation and condensation**

➢ **Fermi gases: formally equivalent when the resonance state has a sufficiently short lifetime [Holland *et al.*, 2004] with pairing function correspondence**

$$< c_{-k\downarrow} c_{k\uparrow} > \Rightarrow < a_{-k\downarrow} a_{k\uparrow} > - \sum_q \frac{g}{2\varepsilon_k - E} \left(b_q a^+_{q+k\uparrow} a_{k\uparrow} - b_{-q} a^+_{-k\downarrow} a_{-q-k\downarrow} \right)$$

$$E = \nu - \sum_k \frac{g_k^2}{2\varepsilon_k - E}$$

➢**Resonance Hamiltonian separates energy scales. Thus advantageous when** $|a_F| \to \infty$ **(no easy way of incorporating energy dependence in single-channel model)**

❑ Theoretical Issues on the Crossover…II

3. BCS & BEC limits

Theories have to reproduce correct BCS and BEC limits

- ✓ **BCS: easy as most calculations start from BCS ground state**

- ✓ **BEC: Petrov *et al* '03 point out that the boson-boson scattering length is $\mathbf{a_B = 0.6a_F}$ from solution 4-body Schroedinger equation** ($\mathbf{a}_F$? r_0 potential range)

❑ Theoretical Issues on the Crossover…II

4. Universality@Unitarity limit

Theories have to cope with the unitarity limit $|a_F| \to \infty$

❑**At resonance thermodynamic properties are expected to be independent of** a_F **as the relevant length scale is the interparticle distance** $\approx n^{-1/3}$ **[e.g. Heiselberg 2001, Ho and Mueller 2003]**

❑**Experiments consistent with the "universal" parameter (**$a_F<0$**)**

$$\beta \equiv \frac{E_{int}}{E_F} \approx -0.25 \text{ over the range } 0.1 < \frac{T}{T_F} < 1$$

But Innsbruck measures –0.68

- **Theories range from** $\beta = -0.56$ **[Carlson *et al.* 2003, Astrakharchik *et al.* 2004 QMC T=0] to** $\beta = -0.67$ **[Baker 1999] to** $\beta = -0.3$ **[Bruun 2004]**

- **Argument is based on a single-parameter model for the interactions and may miss an energy scale. Resonance model better suited to cope in the unitarity limit**

On the BCS side analysis of resonance Hamiltonian suggests that universality holds only for broad resonances [Bruun 2004]

$$g^2 ? \frac{4\pi k_F}{m^2}$$

❑ Open problems - Dynamical effects: released momentum distribution and comparison with exp.

Understanding the problem

- **Experiments measuring *e.g.* condensate fraction, specific heat, momentum distribution,…need expansion + imaging. Expansion has to be free, thus a=0 on a fast scale, measuring "released" quantities. How fast? Slow on the 2-body and fast on the many-body scale**

- **Separation of energy scales in ultracold Fermi gas with Feshbach resonance:**

$$\hbar^2/mr_0^2 \sim 10 \text{ mK} \qquad E_F \sim 1\ \mu\text{K}$$

- **How understand the released momentum distribution from exp?**

- **In the JILA experiment**

1. **Start with weakly interacting Fermi gas at $T=0.12\ T_F$ in trap with $\nu_r=280$ Hz and $\nu_z / \nu_r = 0.071$**
2. **Ramp up interactions to given a (B) at a rate of $(6.5\ \mathrm{ms/G})^{-1}$ and wait 1 ms**
3. **Ramp rapidly down interactions to a=0 (B=209.6 G) at a rate of $(2\ \mu\mathrm{s/G})^{-1}$**
4. **Expand for 12 ms and image**

A time-dependent model

- If nonequilibrium processes are slow on the 2-body and fast on the many-body scale, details of short-range potential are negligible and thus

- Interactions can be accounted for by the boundary condition

$$\left[\frac{rG_A(r,t)'}{rG_A(r,t)}\right]_{r=0} = -\frac{1}{a(t)}$$

for the anomalous density matrix $G_A(r,t) = <\psi_{\downarrow}(r,t)\psi_{\uparrow}(0,t)>$

[For the electron gas: Kimball PRA (1973), Niklasson PRB (1974)]

Recipe:

1. **Use hamiltonian of Fermi gas interacting through the pseudopotential**

$$V(t) = \frac{4\pi a \hbar^2}{m}\delta(\vec{r})\left(\frac{\partial}{\partial r}\right) r$$

2. **Derive equations of motion for normal G_N and anomalous G_A density matrices and use mean-field approximation:**

$$i\hbar \frac{d\tilde{G}_N(r,t)}{dt} = \frac{8\pi\hbar^2}{m} i \,\mathrm{Im}\left(\tilde{G}_A(r,t)[\tilde{G}_A(r,t)]_{r=0}\right)$$

$$i\hbar \frac{d\tilde{G}_N(r,t)}{dt} = -\frac{\hbar^2}{m}\frac{\partial^2}{\partial r^2}\tilde{G}_A(r,t) + \frac{8\pi\hbar^2}{m}\tilde{G}_N(r,t)[\tilde{G}_A(r,t)]_{r=0}$$

$$[\tilde{G}_A{}'/\tilde{G}_A]_{r=0} = -1/a(t)$$

3. Equations are solved evolving the initial equilibrium condition with given a(t=0) up to a final t_f with $a(t_f)=0$ with the parameters as in the experiment

4. The released momentum distribution is determined from

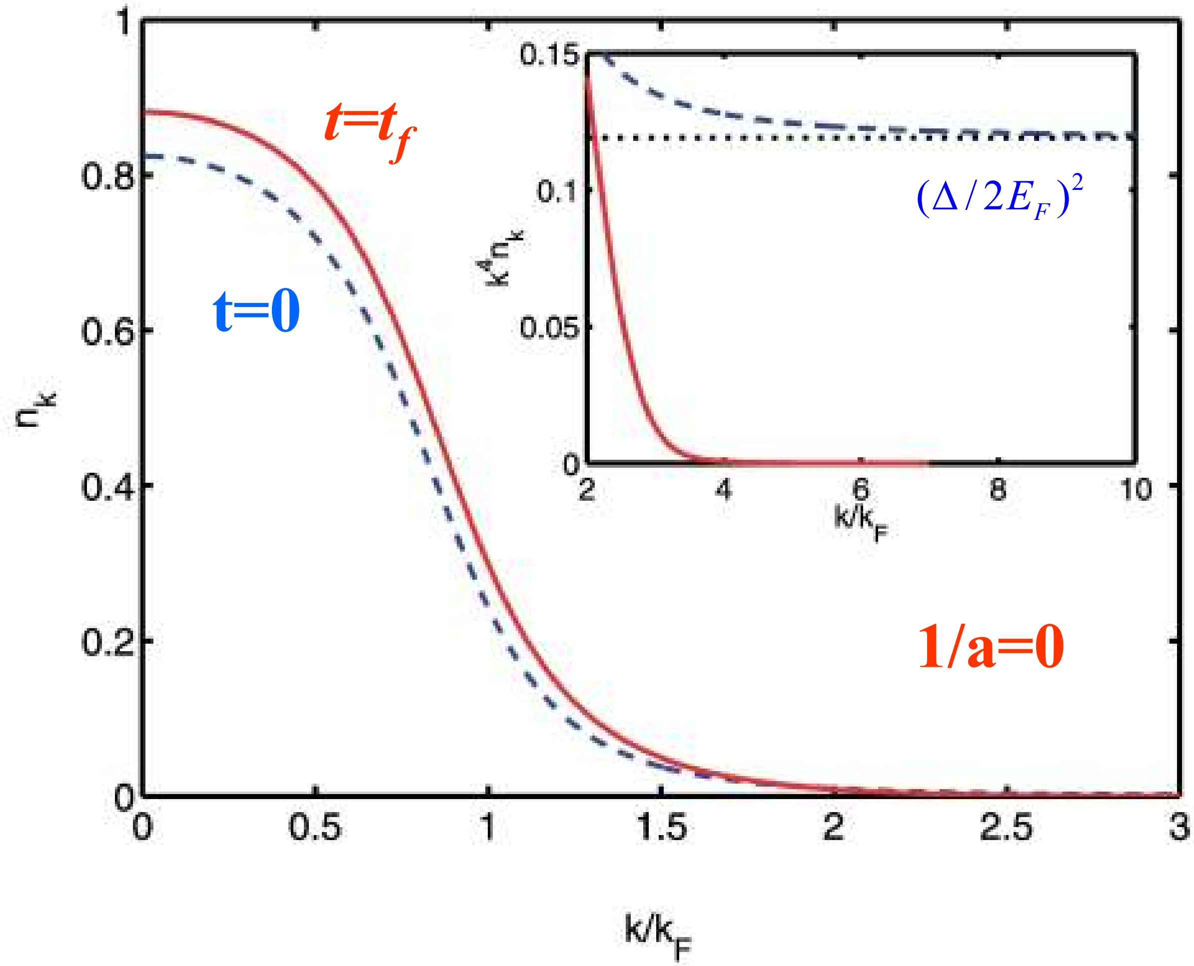
$$n_k(t=t_f) = \int d\vec{r}\, e^{i\vec{k}\cdot\vec{r}} G_N(r, t=t_f)$$

and compared with the experimental data

5. The released energy is determined from

$$E_{rel} = \int d\vec{k}\, \frac{\hbar^2 k^2}{2m} n_k(t=t_f) / \int d\vec{k}\, n_k(t=t_f)$$

➢Homogeneous gas



✓The time evolution kills the large k-tail and makes the kinetic energy finite!

✓ Equilibrium state has the unphysical large k-tail $1/k^4$ arising with zero-range potentials:

Bose gas @ T=0

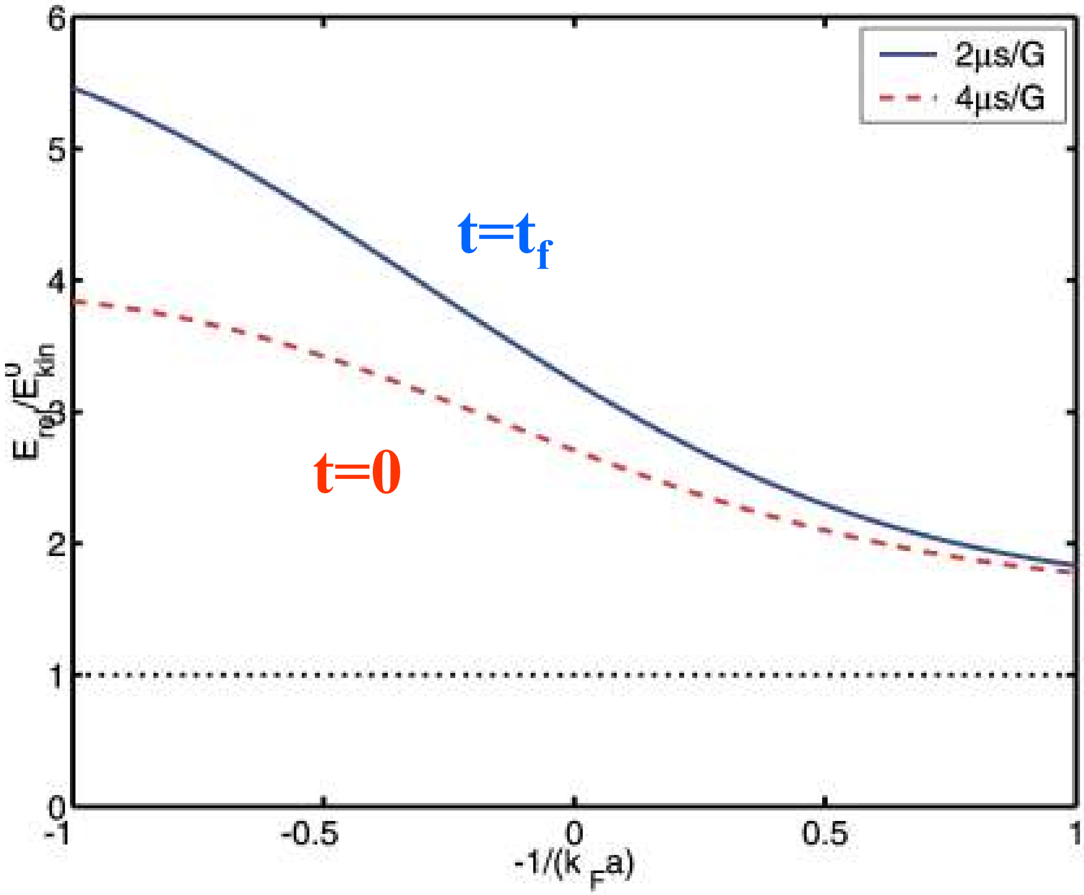
$$k^4 n_k \sim (16(8\pi n a)^2)^{-1}$$

Fermi gas with a<0

$$k^4 n_k \sim (m\Delta/\hbar^2)^2$$

Fermi gas with a>0

$$k^4 n_k \sim (4 a k_F^5 / 3\pi^2)$$



✓On the far BCS side $1/ k_F a(0) >> 1$ the released energy reduces to noninteracting value

$$E^0_{kin} = 3E_F/8$$

✓On BEC side sensitivity to high-energy tails of n_k and thus to ramp rate

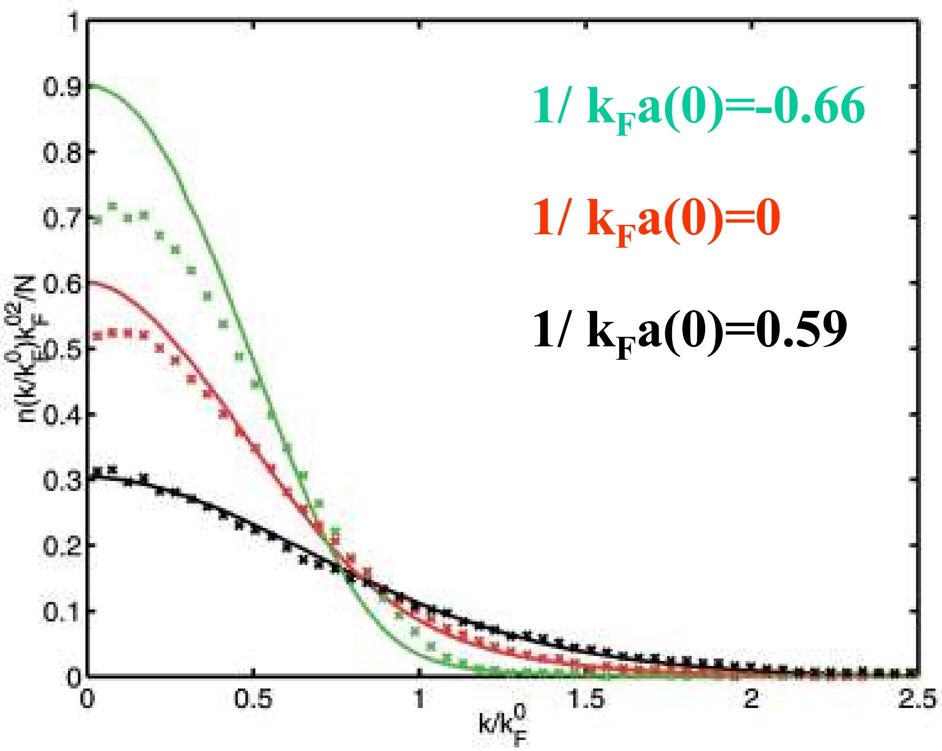
✓On the far BEC side $-1/ k_F a(0) << -1$ the released energy reduces to the dissociation energy of the molecular state

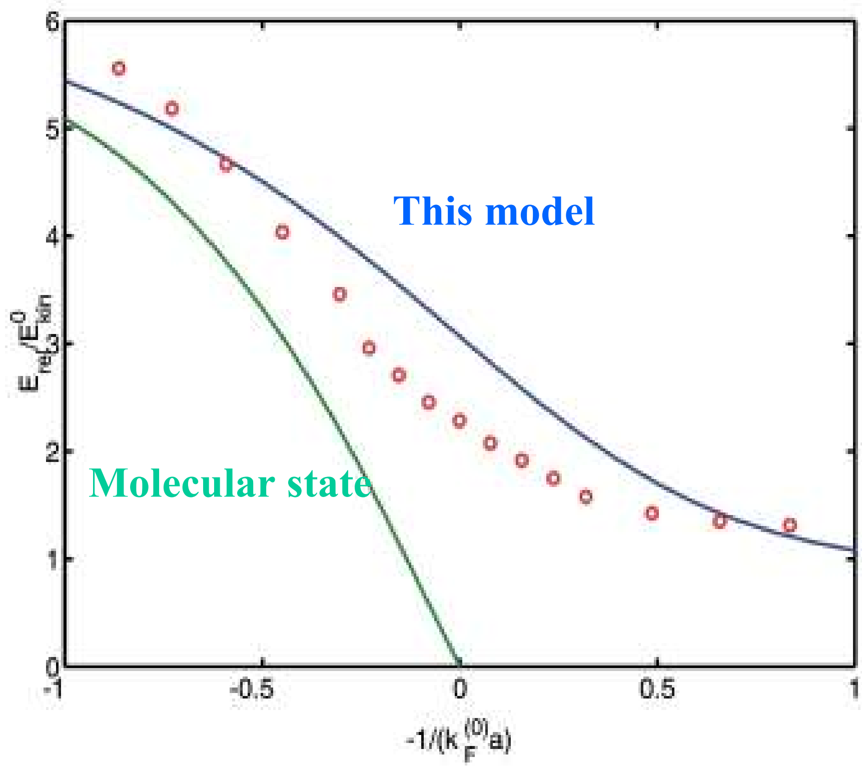
➢Trapped gas:

✓use the Local Density Approximation for equilibrium state

✓ evolve each slice in cm-coordinate R of the density matrices

✓ compare integrated (axial) column densities as in JILA exp





Message

- The time-dependent model we have developed clarifies that role of dynamical effects in the ramp is to suppress the high-energy tail of the momentum distribution

- The model reproduces qualitatively the JILA experimental data by Regal *et al.* with no fitting parameters

- Quantitative discrepancies:

    - **Deep BCS side:** are due to the Hartree term we have neglected within the present NSR approach

    - **Deep BEC side:** may be due to the finite temperature effects (here T=0). On the exp side, variations of T during the ramp (BEC side) and ramping not fast enough on the axial time scale of the trap
    - **Resonance:** likely due to inadequacy of NSR approach

❑ Open problems - Universality

➢ **Needed a model potential able to:**

✓ **reproduce scattering properties (according *e.g.* to full coupled-channel calculations or experimental data)**

✓ **independently tune overlap of closed to open channel while fixing position from resonance**

✓ **possibly avoid renormalization issues (no contact potential)**

Accurate account of the interactions required ➡

➢ **Eventually want to resort to QMC Simulations of the Fermi gas with Fano-Feshbach resonance**

➢ **Proceed along the steps:**

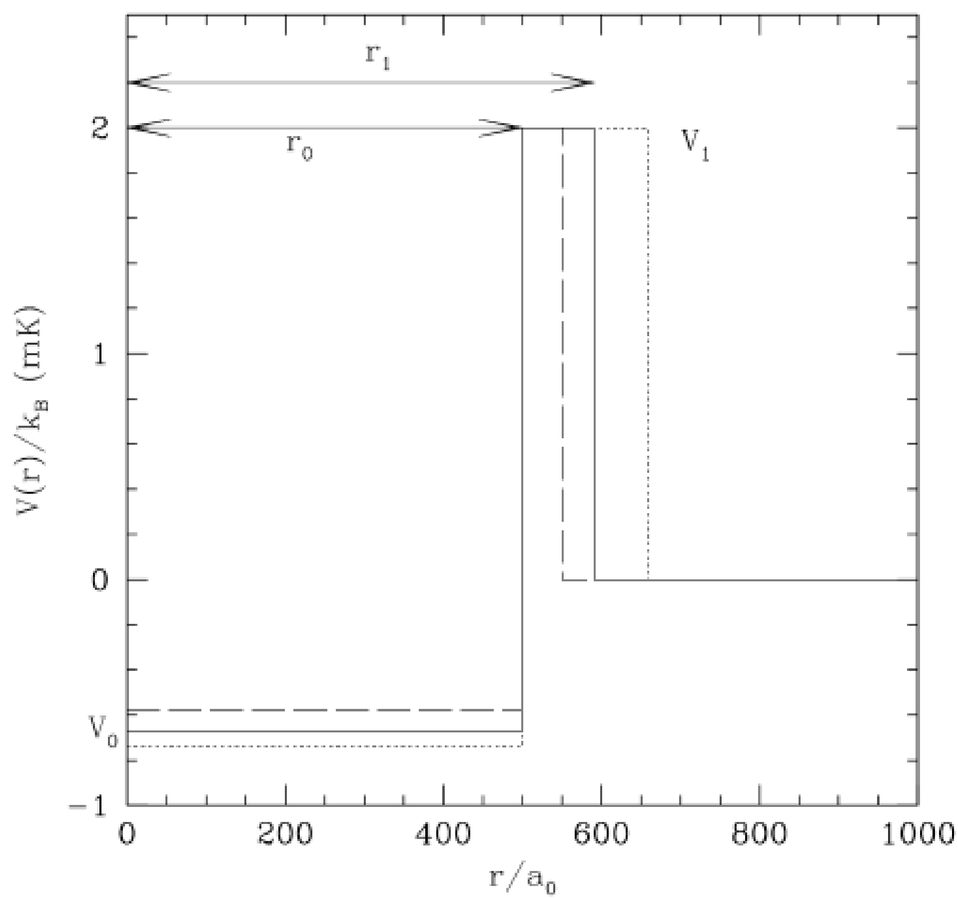
I: Definition of the well-barrier model

II: Mean-field (BCS) ground-state with well-barrier model

III: Quantal Monte Carlo Simulation results

Universality I: Well-barrier model

The well-barrier model for the Fano-Feshbach resonance



- **Conditions**

  $nr_0^3 = \ 1$ diluted

  $na^3 > 1$ unitarity limited

- **Regimes (tunable by this model)**

  $\frac{g\sqrt{n}}{E_F}$ >1 “broad” resonance
  <1 “narrow” resonance

- **Parameters**

$r_0 = 500 a_0$

$a = +5000 a_0$

$n = 1.054 \times 10^{14} \ \ cm^{-3}, \quad nr_0^3 \ ; \ \ 2 \ 10^{-3}$

Fixing *a* while varying $g\sqrt{n}/E_F$

❑ 2-body T-matrix

$$T(k) = \frac{2\pi\hbar^2 i}{mk}\left[S(k) - 1\right]$$

with

$$S(k) = e^{-2ika_{bg}}\left[1 - \frac{2ik|g|^2}{-\frac{4\pi\hbar^2}{m}\left(\nu - \frac{\hbar^2 k^2}{m}\right) + ik|g|^2}\right]$$

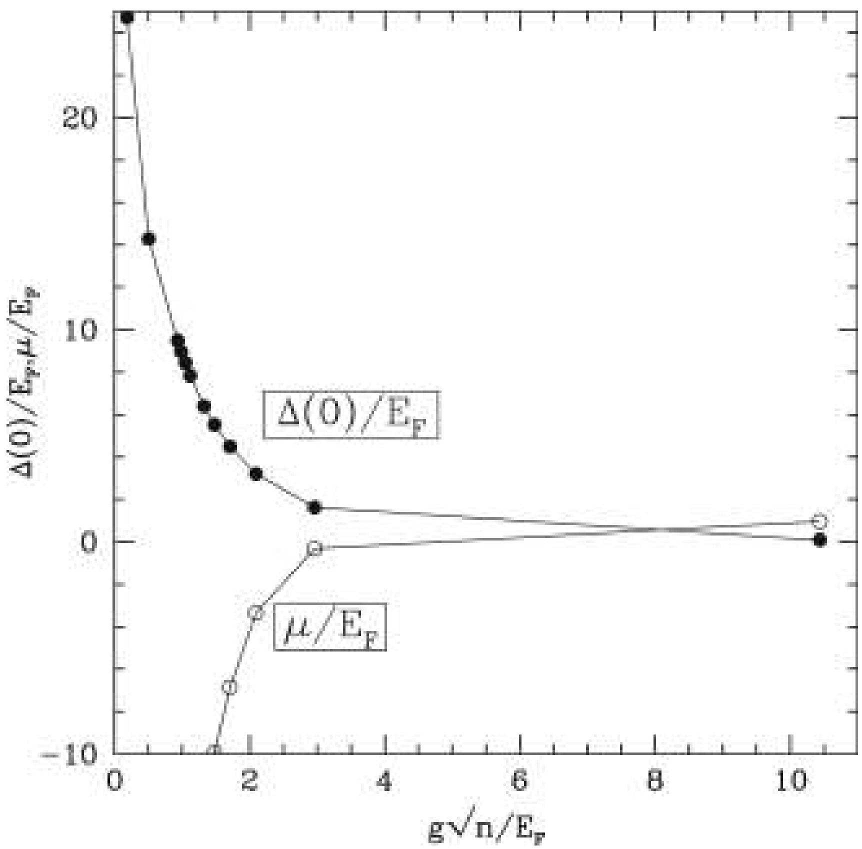
❑ Extracting Properties

$$\frac{4\pi h^2 a}{m} = \lim_{k\to 0} T(k) \qquad |g|^2 = -\frac{8\pi\hbar^4}{m^2 R_{eff}}$$

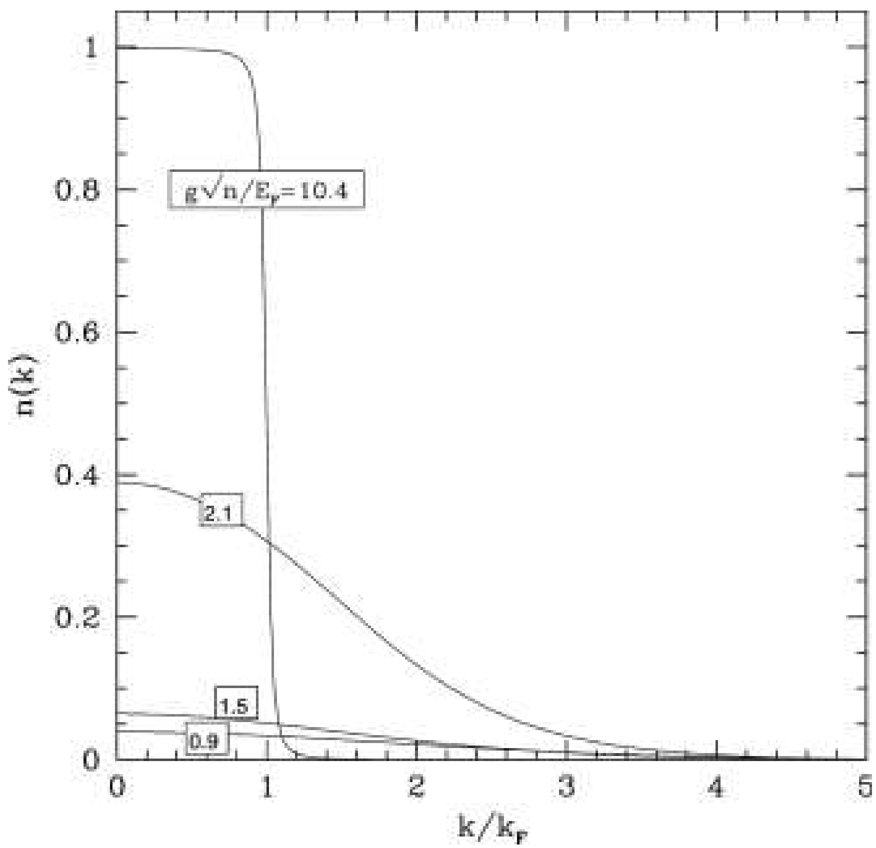
$$R_{eff} \equiv -(4\pi\hbar^2/m)(d^2 T(k)^{-1}/dk^2)|_{k=0}$$

Effective range

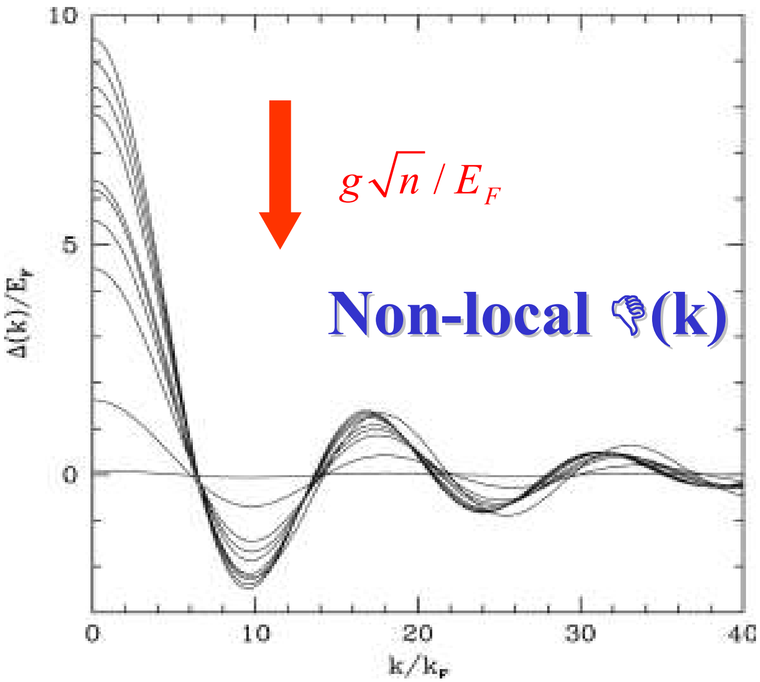
Universality II: Structure and Thermodynamics of the fluid at the mean-field level – BCS solution

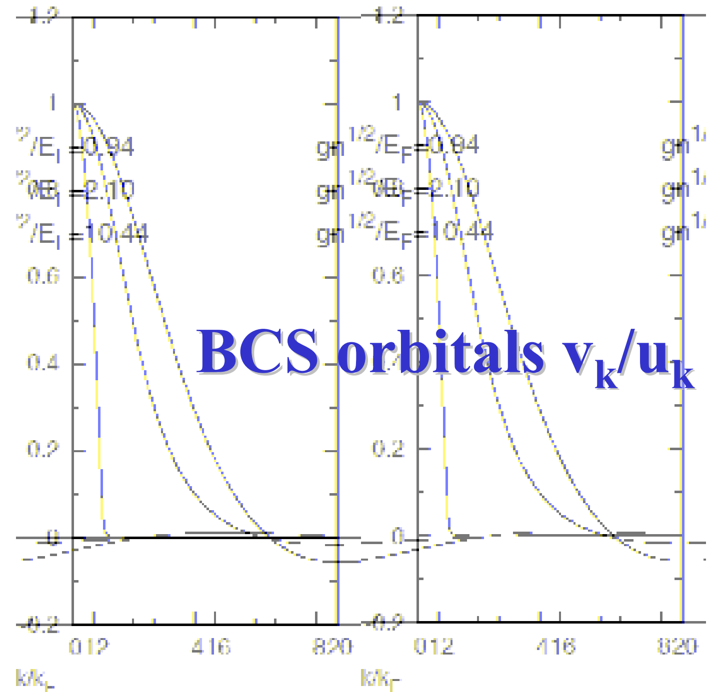


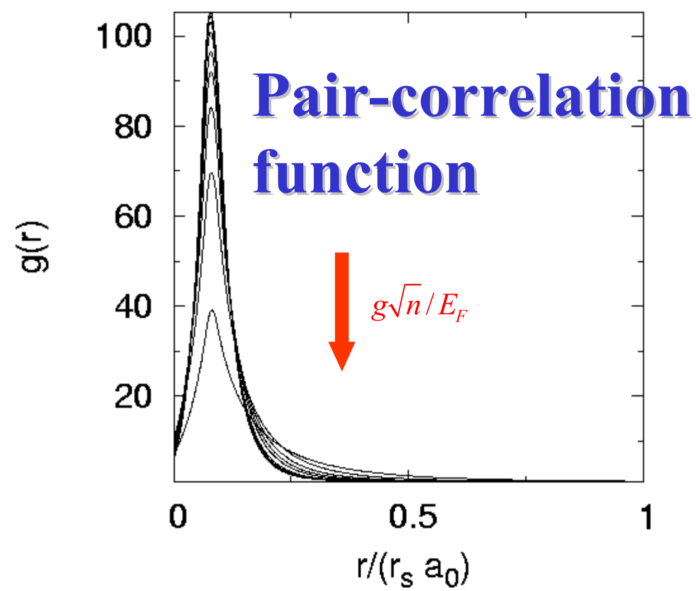
Gap and chemical potential



Momentum distribution







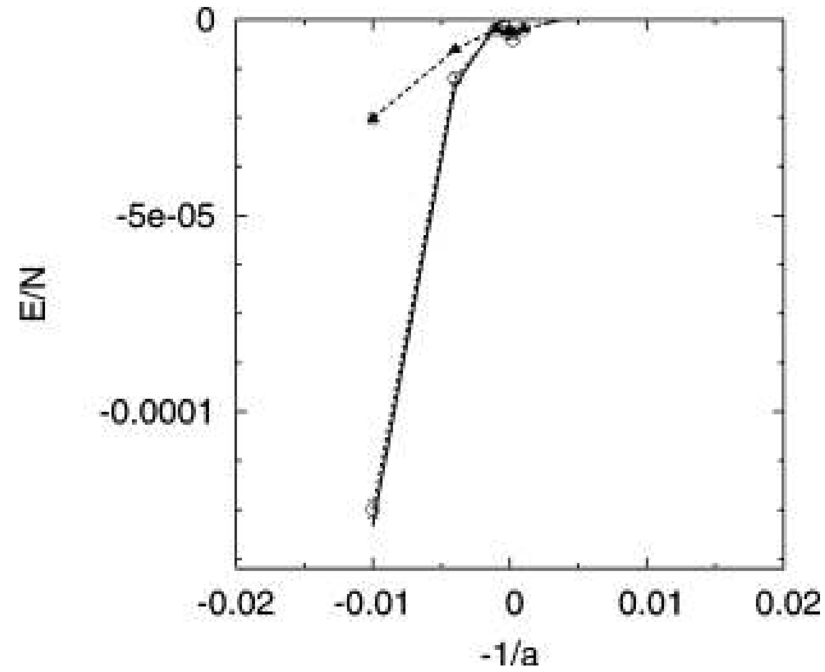
$$g(r) = g_{HF}(r) + g_p(r) \, ,$$

$$g_{HF}(r) = \frac{1}{4} - \left(\frac{1}{(2\pi)^3 n}\right)^2 \int d\mathbf{k}\, e^{-i\mathbf{k}\cdot\mathbf{r}} |v_\mathbf{k}|^2$$

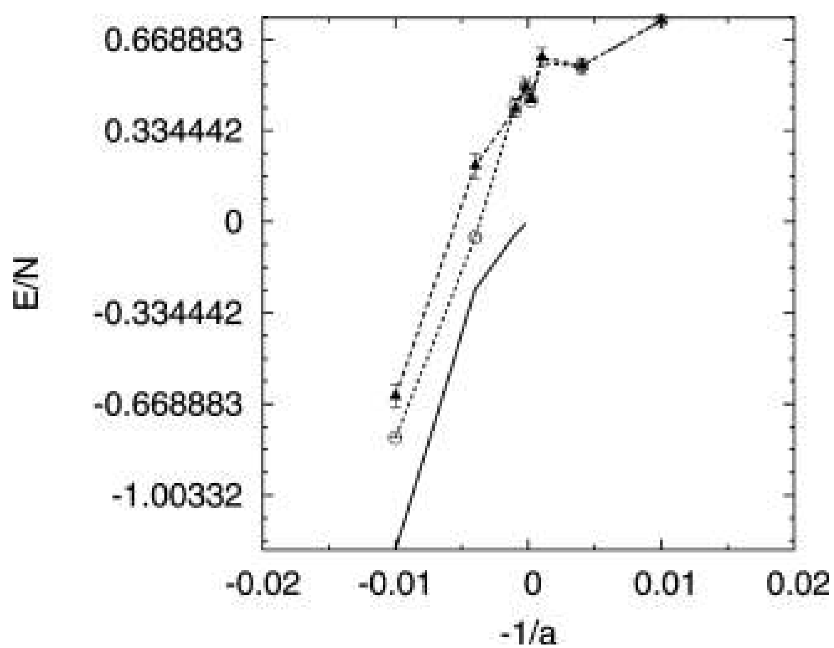
$$g_p(r) = + \left(\frac{1}{8\pi^3 n}\right)^2 \int d\mathbf{k}\, e^{-i\mathbf{k}\cdot\mathbf{r}} u_\mathbf{k} v_\mathbf{k}$$

Universality III: Preliminary Quantum Monte Carlo results

- Parametrize the well-barrier model to describe the scattering properties of the broad and narrow resonances of 6Li.
- Use of BEC, BCS, and Plane Wave trial wavefunction (with or without Jastrow) to try different nodal structure approx. – that can be crucial with Fermi systems [as in Carlson *et al*.]
- Plug the trial wf into a Variational MC (optimization is in some cases needed)
- Determine the "ground-state" (within the nodal approximation used) by means of Reptation QMC [Baroni&Moroni 1998]
- Calculate energy/particle E/N, momentum distribution, pair-correlation function and the structure of the fluid in the crossover and compare broad and narrow resonance results



Preliminary RMC data
Broad resonance 6Li



Preliminary RMC data
Narrow resonance 6Li

Message

- **The dependence of the scattering phase shift on energy significantly affects the thermodynamic properties @ resonance**

- **A BCS superfluid emerges on the BEC side of the resonance (a>0) while the quality of the resonance increases**

- **Non-universal behaviour is found on the BEC-side of the resonance, depending on the quality of the resonance**

- **Variations of thermodynamic properties are smooth on the scale of the Fermi temperature, indicating bosonic-like character**

- **Preliminary QMC data suggest that NSR nodal approximation to the ground state is not sufficient to describe the Fermi gas in the presence of narrow Feshbach resonance**

Current and Future…

Atomtronics with Quantum Resonant Tunneling-based Devices

[NJP03,J.Mod.Opt.04,J.Op.B05]

Proposal submitted @DOE (ext. collaborator), in the course of referral from LANL
Collaborators: A. Smerzi@LANL, M. Artoni@Brescia and G.C. Larocca@SNS

Current and Future…

- **Atomtronics is an emerging paradigm aimed at storing, guiding and building devices with atomic waves, as electrons in nanostructures, spins in spintronics and photons in photonics**

- **Advantage of using BECs is in achieving better focussed-beams with long coherence length and short "wavelength" (a few tens of nm) and efforts are currently and successfully pursued to put BECs in chips**

- **Applications of atomtronics circuitry are *e.g.* in precision measurements of fundamental constants, tests of fundamental forces (*e.g.* Casimir-Polder) and principles, interferometry, atom lithography (optical litography is bounded to 100 nm), quantum computation**

- **Quantum Resonant Tunneling can be the building block to develop a new generation of atomtronic devices, exploiting its (i) multifunctionality, (ii) high control, versatility, and engeneering capability, (iii) all-atomic conception**

- **Time-dep. tunnelling has long history and is also relevant for**

➡ ***Applications:***

* *photoinduced dynamics in strong laser fields* [Gavrila 1992]

* *high-frequency field impurity ionization* [Ganichev et al. 2002]

* *transport in superlattices under THz fields* [Guimaraes et al. 1993]

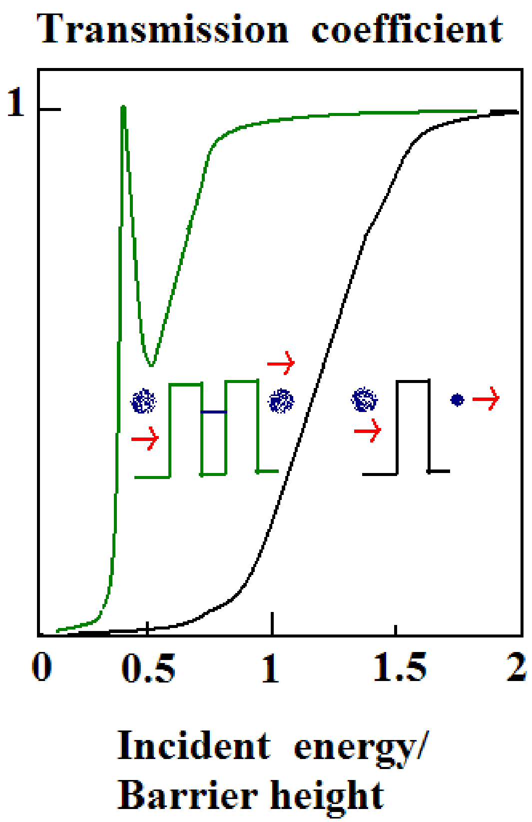
* *quantum chaos* [Hensinger *et al*., Steck *et al*. 2001, Averbukh *et al*. 2002]

* *diffusion and relaxation processes* [Doering-Gadoua 1992].

➡ ***Unravel fundamental concepts in Quantum Mechanics:***

* *the controversial notion of tunnelling time* [Buttiker-Landauer 1982].

- Resonant tunnelling across static double barriers manifests as a peak in the transmission of a wavepacket at energies resonant with the quasi-bound state inside the double barrier, well below the threshold for tunneling across the single barrier



- Resonant tunnelling is also possible when particles move across time-dependent external potentials. For example

$$V(x,t) = U(x - lf(t))$$

- Nearly perfect transparency has been predicted for charged particles in oscillating laser fields [Vorobeichik *et al*. 1998, Pimpale *et al*. 1991, Ge and Zhang 1996] but never observed!!!!!!!!!!!!!!!!!!!!!!

Goals

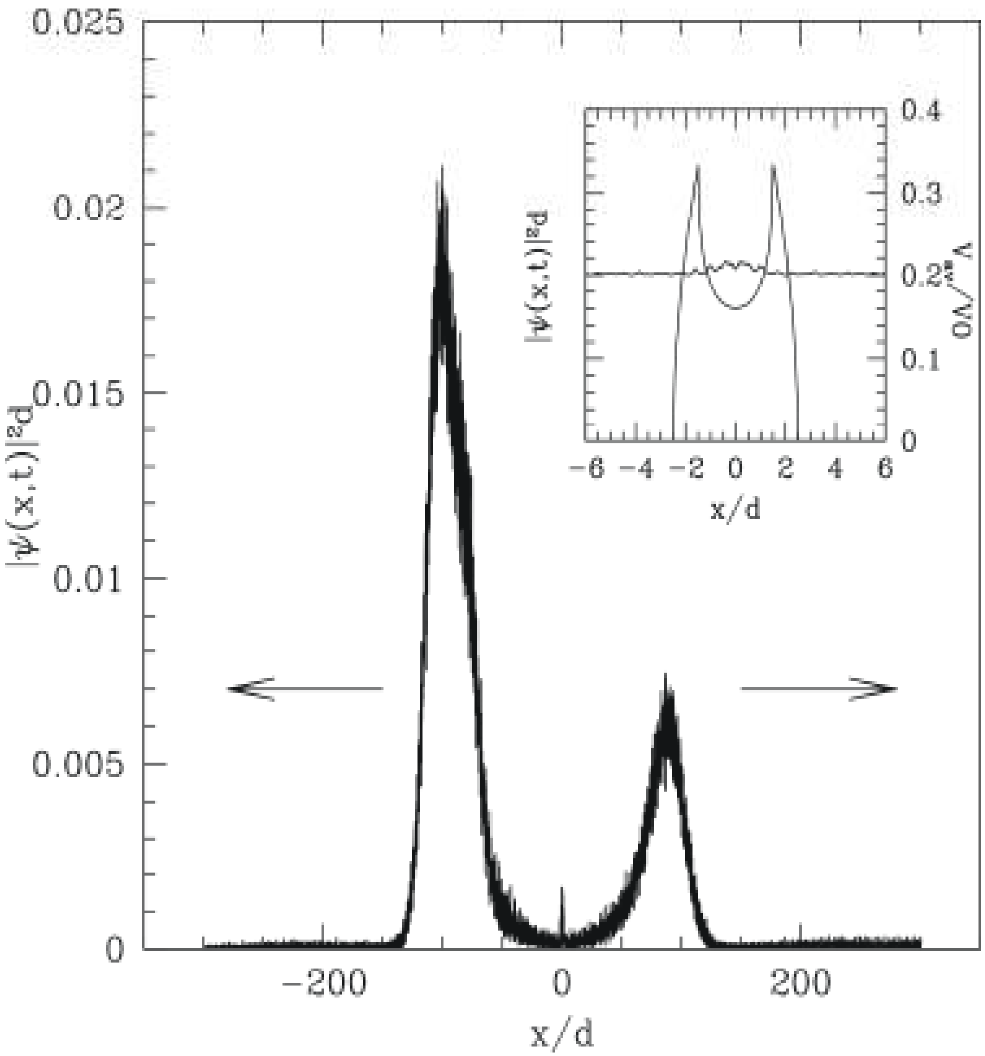
We predict that the use of ***ultracold atomic beams*** impinging on repulsive dipole potentials set spatially oscillating at high frequencies, *e.g.* modulated via an oscillating mirror [Anderson and Kasevich 1998, Burger et al. 2001, Cataliotti et al. 2001, Greiner et al. 2002]. yields access to the ***observation*** of :

- ***Almost perfect field-induced transparency at tunable energies*** at high frequencies [Chiofalo, Artoni and La Rocca 2003].
- ***Energy filtering of the atomic beam*** and ***generation of atom-laser sidebands*** in the so-far unexplored ***crossover region*** corresponding to intermediate frequency driving [Chiofalo, Artoni and La Rocca 2004].
- Possibly, useful effects for atom-optical devices such as ***optical limiting*** and ***optical bistability*** after exploiting nonlinear atomic interactions at high frequency driving [Embriaco, Chiofalo, Artoni and La Rocca 2004]

Analyze one effect at a time

- Start with the case of noninteracting atoms at high frequency driving
- Proceed by tuning the frequency from very low to high
- Go back to high frequencies but switching on the nonlinear interactions

The Physical Mechanism



❖An atomic beam with $E_k = 0.2V_0$ near resonance and spread ΔE impinges on a light barrier of width d and height V_0 oscillating as

$$V(x,t) = V(x - l\ \cos(2\pi\nu\ t))$$

$$V(x) = V_0[\theta(x - d/2) - \theta(x + d/2)]$$

$$\nu = 10\ KHz \text{ and } l = 2d$$

❖A large portion of matter wave is transmitted well below barrier and a fraction still dwells inside the barrier

❖Why (strictly true for $\nu \to \infty$): the time-averaged potential $V_{av} = 1/T\int V(x,t)dt$ allows for metastable states with energy E_0 and width Γ_0 possibly resonant with the incoming beam.

The Model

❑ **The problem can be considered as <u>one-dimensional</u>, since in the experiment the atomic beam travels along a <u>waveguide</u>. If the transverse confinement energy is much larger than all other energies, the wavefunction in the transverse direction is frozen**

❑ **One-dimensional Gross-Pitaevskii equation, that is a nonlinear Schroedinger equation for the condensate wavefunction** $\psi(x,t)$ **along the direction of motion x**

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t}=\left[-\frac{\hbar^2\nabla^2}{2m}+V(x,t)+g\left|\Psi(x,t)\right|^2\right]\Psi(x,t)$$

moving under the action of the barrier V(x,t) oscillating at frequency ■ (or the static averaged potential V_{av}(x)) ν

❑ **Atomic interactions** **enter the nonlinear term, with g modelled from the 3D value** $g^{3D} = \frac{4\pi\hbar^2 aN}{m}$ **to maintain the same level of average interactions**

$$E_I = \frac{\int g \left|\Psi(x,t)\right|^4 dx}{\int \left|\Psi(x,t)\right|^2 dx} = \frac{\int g^{3D} \left|\Psi^{3D}(\vec{r},t)\right|^4 d\vec{r}}{\int \left|\Psi^{3D}(\vec{r},t)\right|^2 d\vec{r}}$$

For a gaussian wavepacket of oscillator length $a_\perp$ **and frozen in transverse direction the condition is satisfied by**

$$g = \frac{g^{3D}}{\pi a_\perp^2}$$

Requirements and System Parameters

- Requirements to observe a clean effect:
  - ✓ $l > d$ to have double-barrier structure in V_{av}
  - ✓ V_0 and l to have ony one quasi-bound state in V_{av}
  - ✓ $\Delta E = \ \Gamma_0$ (thus need to use a BEC)
  - ✓ ν ? inverse tunnelling time to have high transparency

- **System parameters are here tailored for sodium atoms from a transfer-matrix calculation and can be rescaled for other species**

d	V_0/h	E_0/h	Γ_0/h
$827nm$	$4.2KHz$	$0.90KHz$	$0.06KHz$

- **For the noninteracting atomic beam we have taken**

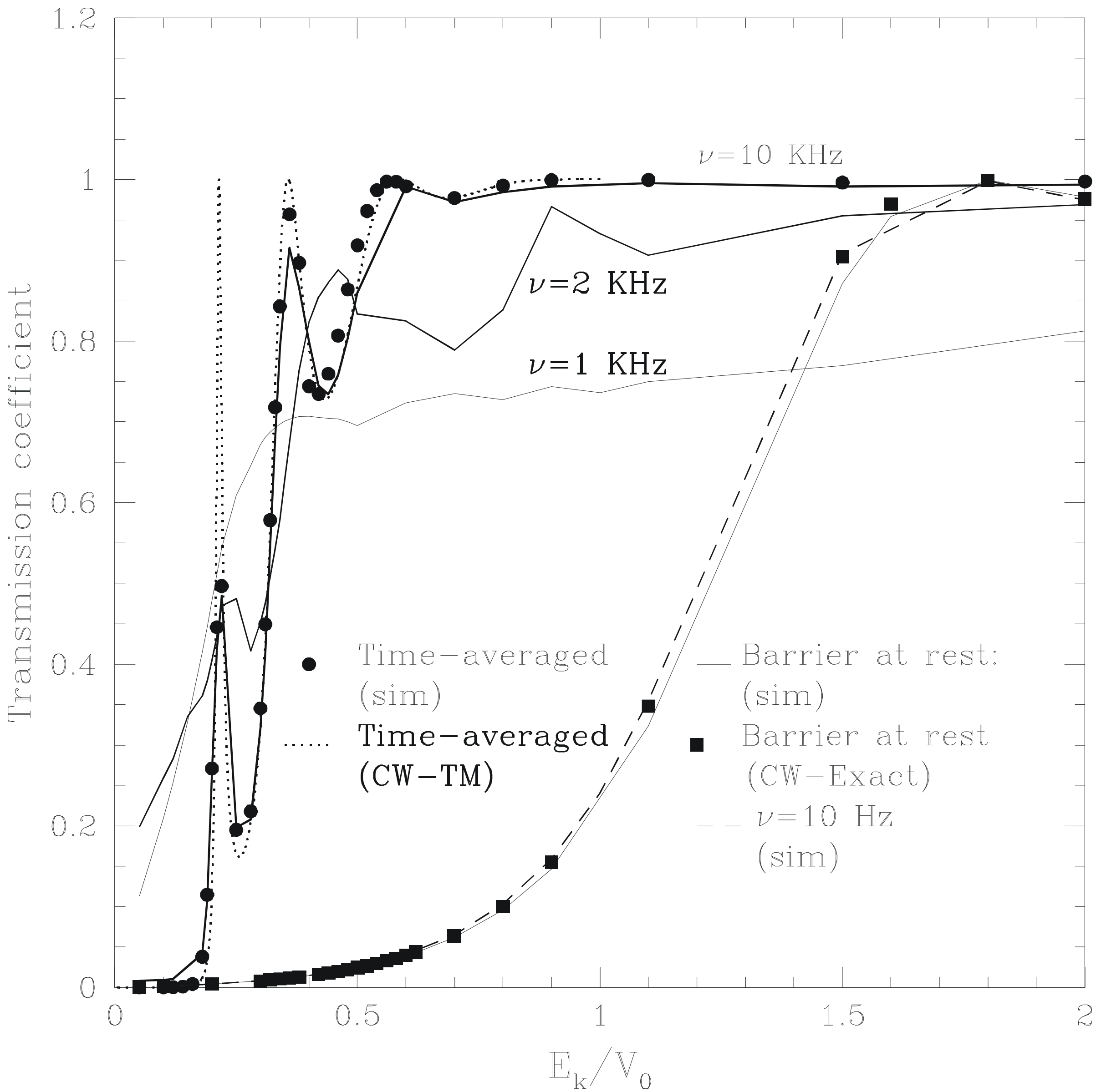
$$\Delta E/h = 0.21KHz$$

- **Reasonable set of parameters for the interacting atomic beam**

$$\Delta E^{*}/h;\ 0.016\,KHz = 0.0039\,\mathrm{V}_0 \qquad \mathrm{g};\ 2\mathrm{g}_0 \equiv 2\frac{\mathrm{h}^2}{2md} \qquad a_{\perp};\ 1\,\mu m \qquad \mathrm{N}=100$$

- **Atomic velocities:** 2 **to** 18 **mm/s** $i.\,e.\, 0.1 < E_k < V_0 < 1$

Results I: General Layout and $\nu \to 0$, $\nu \to \infty$ limits

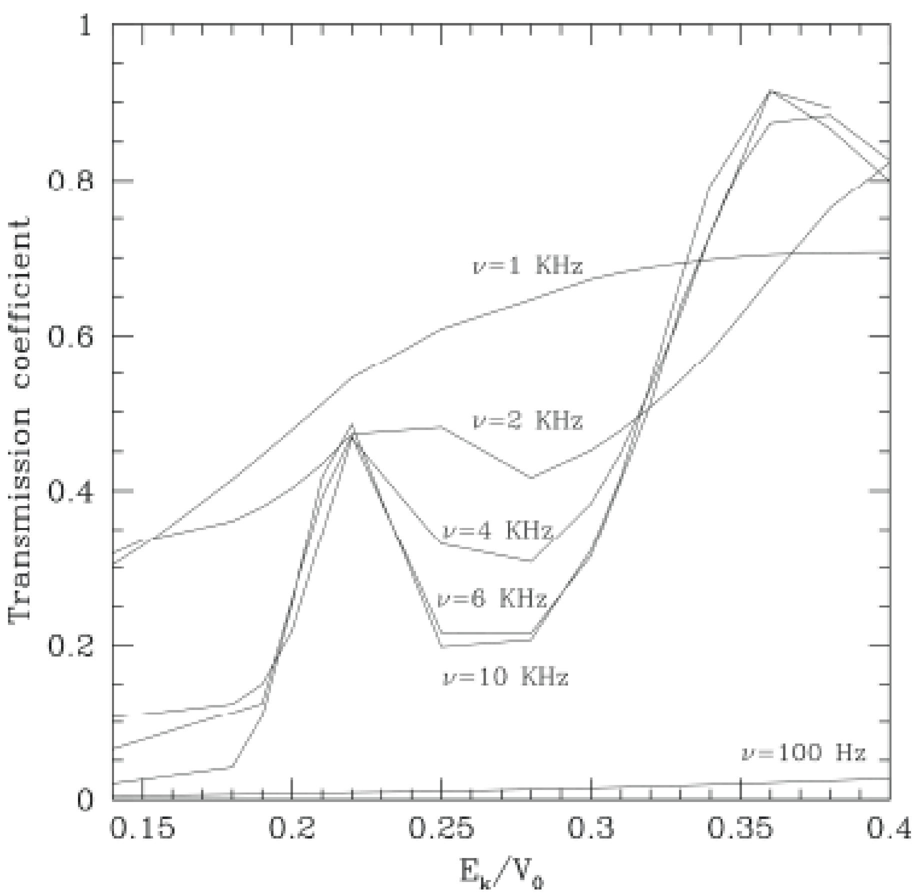


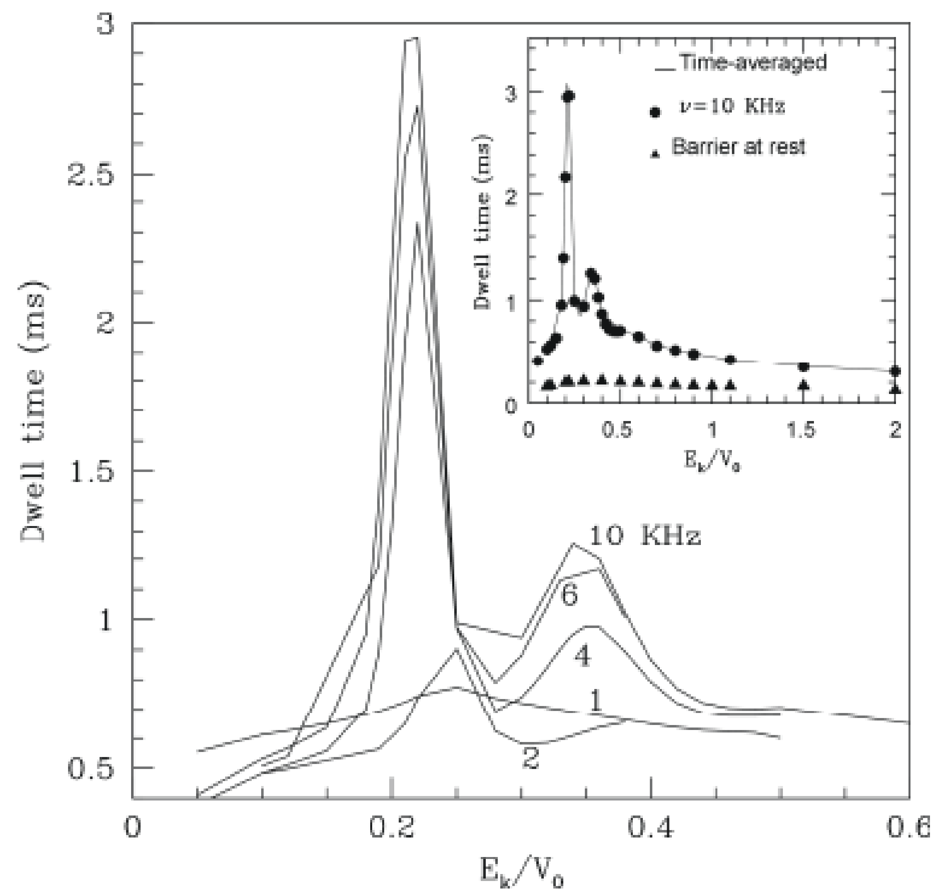
High-frequency limit:
(@$\nu = 10\ KHz$ here)
full transparency well below threshold as seen in tunnelling across V_{av}

Low-frequency limit:
(@ $\nu < 500\ Hz$ here)
Doppler shift depending on the initial phase for
$100\ Hz < \nu < 500\ Hz$

Results II: Crossover Region

- Onset of resonant tunnelling peak occurs with increasing ν
- accompanied by a pronouncing sharp maximum in τ_D
- τ_D very sensitive to changes in ν

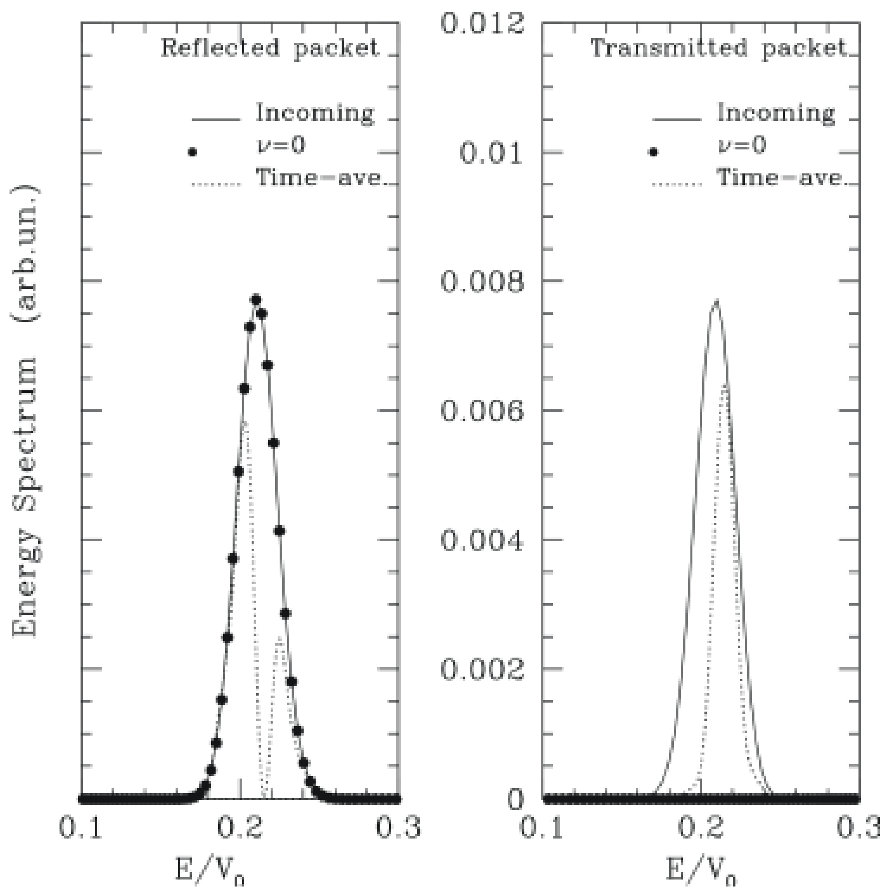




Results III: Elastic vs. Inelastic Processes

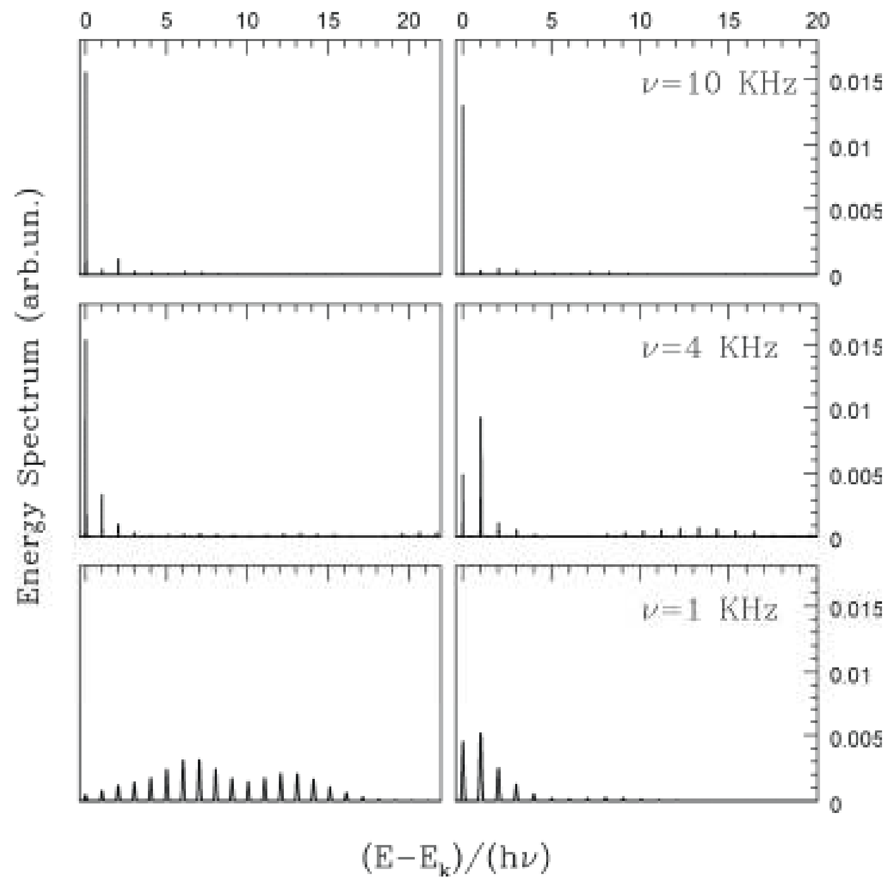
- Inelastic processes signalled by additional peaks at multiples of ν
- Sideband amplification in the crossover region

Energy Spectrum @ High and Low Frequency



Reflected **Transmitted**

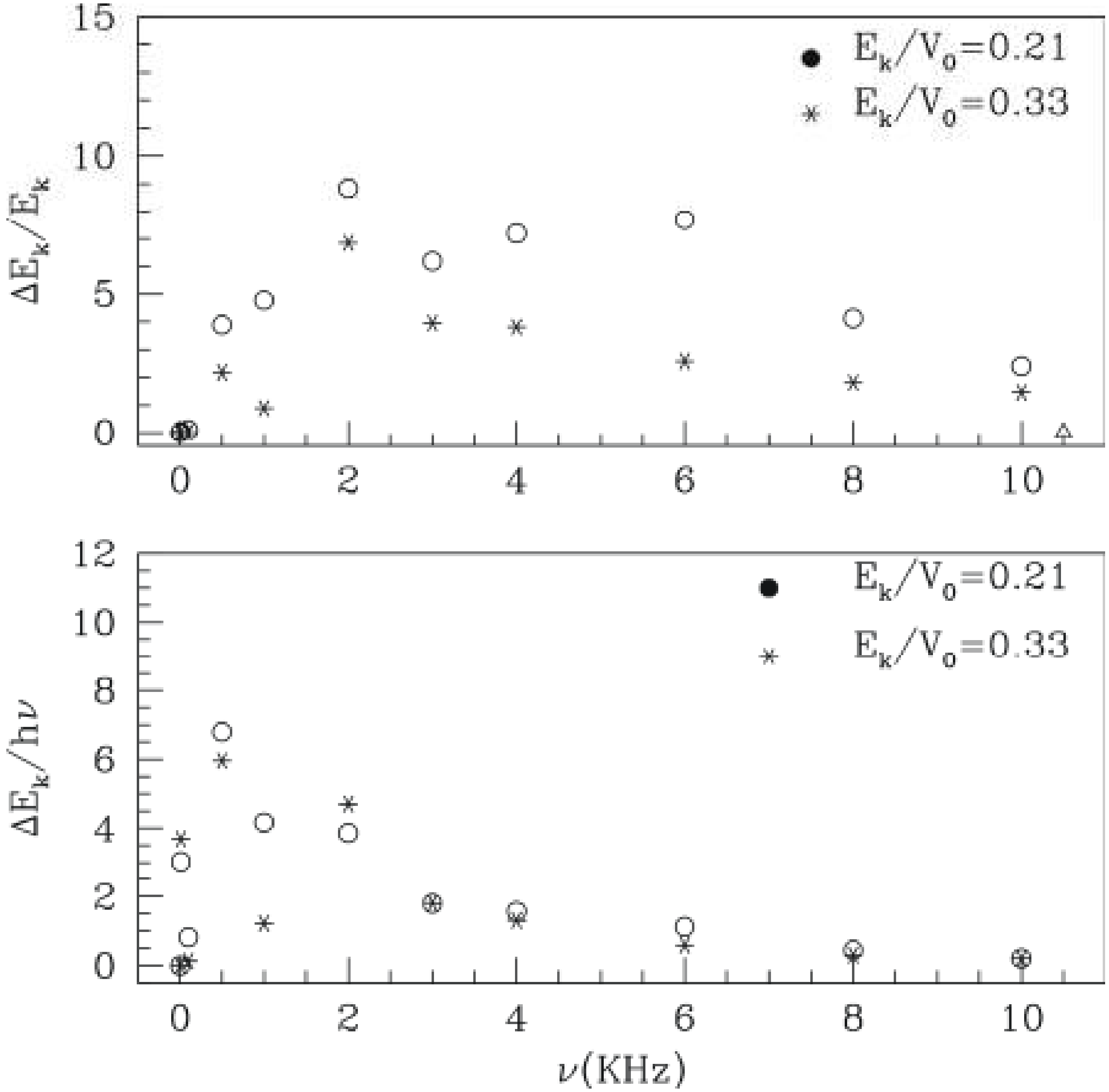
Energy Spectrum @ Finite Frequency



Reflected **Transmitted**

Results III: Elastic vs. Inelastic Processes

- Energy loss vs. frequency

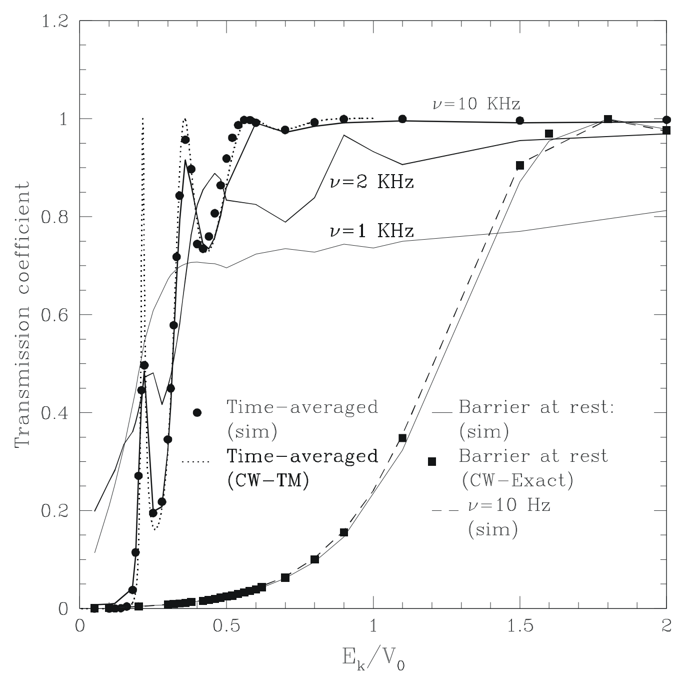


Close to resonance

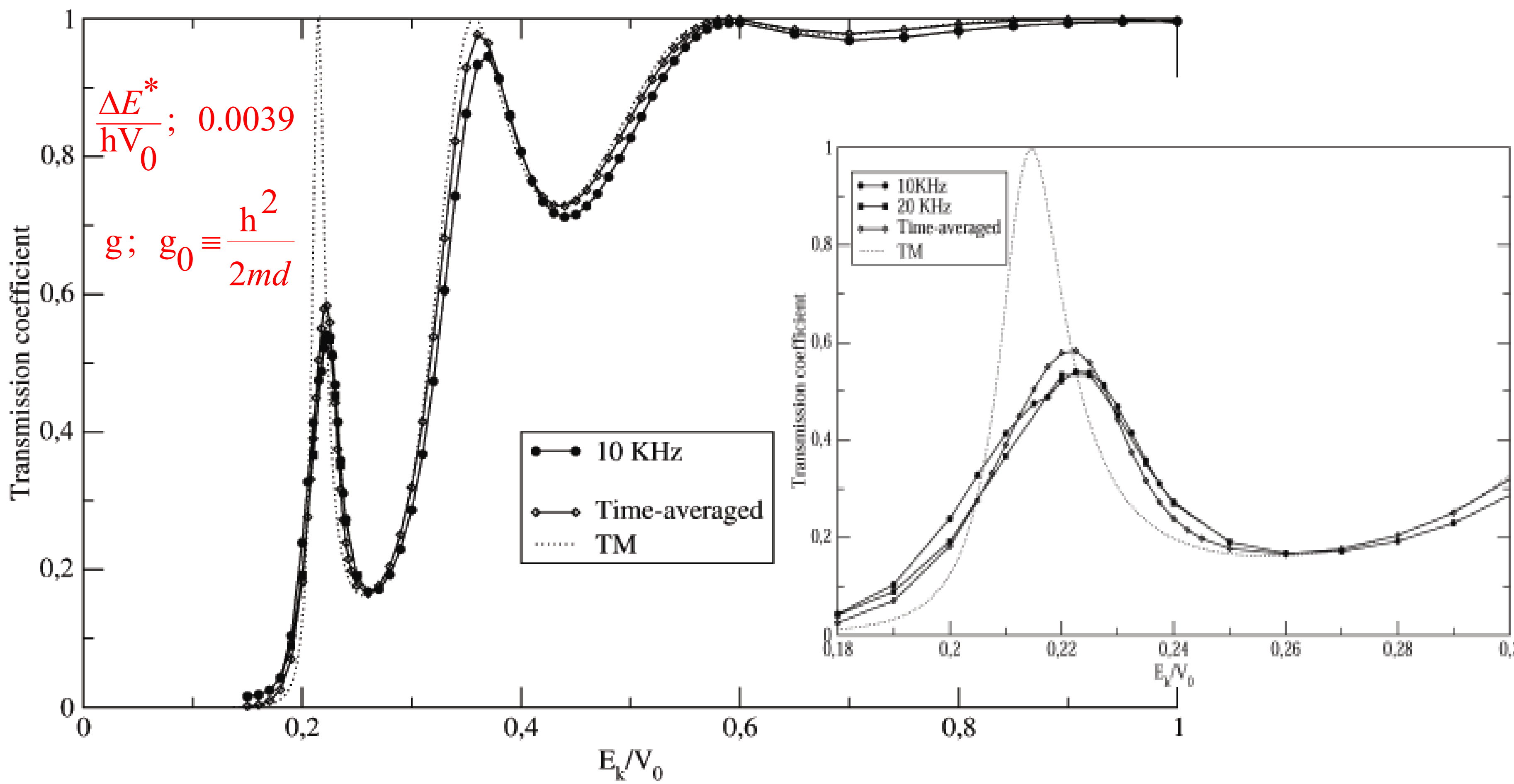
$$E_k / V_0 = 0.21$$

Half-way to continuum

$$E_k / V_0 = 0.33$$



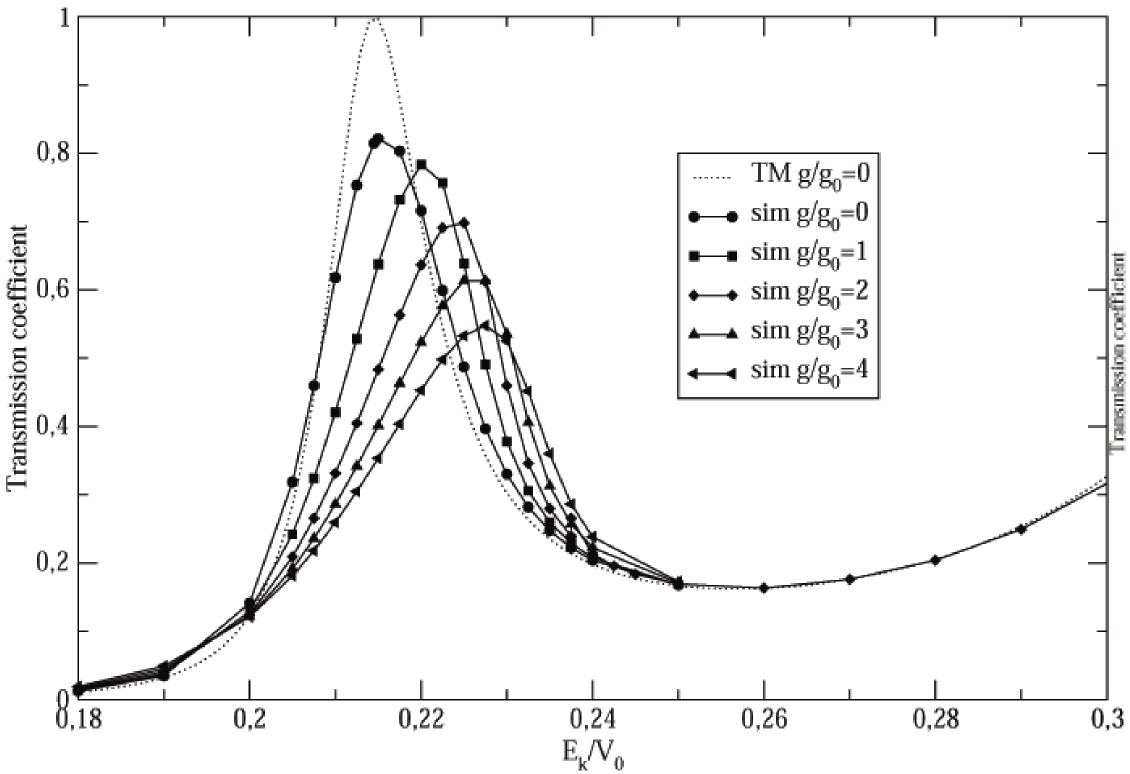
Results IV: Effect of atomic interactions

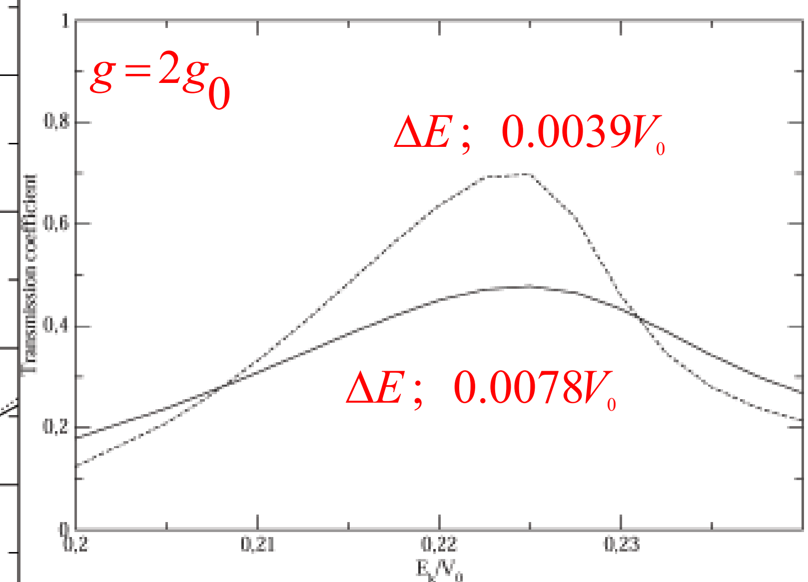


- Interactions preserve the peak provided ΔE be sufficiently small
- Agreement of time-averaged and time-dependent data is worse

- Atomic interactions shift the position of the peak because position of metastable level in V_{av} is shifted

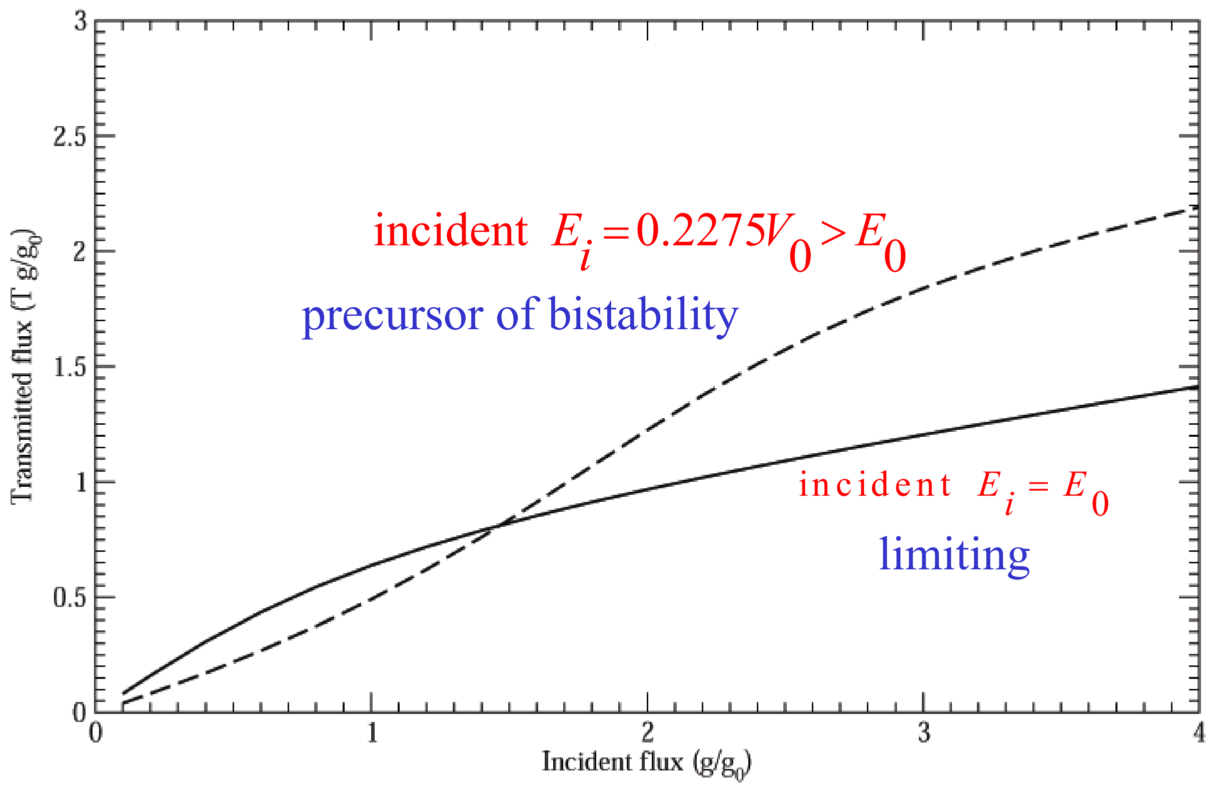
- Atomic interactions flatten the peak because interactions increase the energy width of atomic wavepacket





❑ **Atom-optical limiting occurs when output signal saturates with increasing incident flux (visible when $E_i = E_{0,}$ solid curve)**

❑ **Atom-optical bistability manifests as enhancement and then supression of output signal saturates with increasing incident flux (visible when $E_i > E_{0,}$ dashed curve). Here, only precursor ☹**



$g/g_0 \propto N$

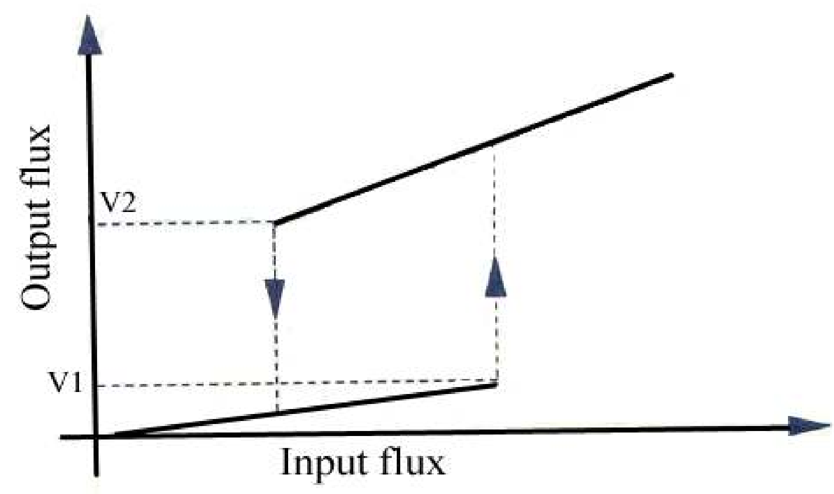
- Which 2- and 3-ports devices can be built up from our QRT?

  - **Filtering and related devices:** coherent reshaping of atomic wavepackets (2P), rectification of energy-modulated input beams (2P), generation of energy-modulated output beams (3P)

  - **Side-bands generation and amplification and related devices:** pulsed atom laser, tunable energy atom comb, entanglement and side-band addressing (3P)

  - **Beam-splitting:** 1D and Y (2P)

  - **Atom-optical limiting and bistability, and related effects:** matter-wave switch, logical gates, tunable beam-splitters (3P)



Test of the Equivalence Principle with Quantum Degenerate Atomic Gases

[PLA03, PLA03, PLA03, Rev. Sci. Inst.06, Rev. Sci. Inst.06]

Proposal submitted to ASI (coordinator) within a global proposal on space experiments for fundamental physics

Collaborators: R.Onofrio@Dartmouth, L.Viola@Dartmouth, M.Artoni@Brescia, G.Larocca@SNS, A.Nobili@Pisa, G.Tino@Florence

➢ Precise gravitation measurements are one of the keys for testing the foundations of cosmological theories and of the general relativity

➢ Crucial concept is the Universality of Free Fall: all bodies fall with the same acceleration regardless of their mass and composition

➢ First experimented by Galileo with a 10^{-3} accuracy and then formulated by Newton in terms of equivalence between inertial and gravitational mass, UFF leads to the Weak EP at the base of General Relativity: gravitational field and accelerated ref frame are *locally* equivalent

➢ Landmark experiments measuring differential accelerations are by Eotvos ($\eta \cong 10^{-9}$) with a torsion balance, improved by Dicke and Braginski ($\eta \cong 10^{-10}$) and Adelberger ($\eta \cong 10^{-12} - 10^{-13}$)

Current and Future…

- **Space experiments in low-Earth orbiting satellites would improve the accuracy by up to 4 orders of magnitude (larger driving signal) bringing the accuracy to the level where EP-violations are predicted**

- **Use of quantum objects instead of macroscopic bodies would be a major achievement, as the definition itself of EP requires some care [Viola&Onofrio]**

- **Ultracold atomic gases can be manipulated to accommodate different quantum states, and their dynamics can be controlled with high accuracy and are thus ideal candidates to test EP in the quantum world, with the perspective of linking gravitation and quantum mechanics**

Current and Future…

- **Atomic fountains have been already realized, achieving ($\eta \cong 10^{-10}$) [Chu] on the ground**

- **Experiments have been performed to measure te Casimir-Polder forces with BECs close to surfaces [Cornell, JILA]**

- **Submitted to ASI a project within a larger proposal, also in collaboration with Lorenza Viola and Roberto Onofrio to device a space experiment (longer time-of-flight and higher sensitivity in the preparation of initial different quantum states) to test the EP and study the deviation from Newtonian gravity on a micro-scale**