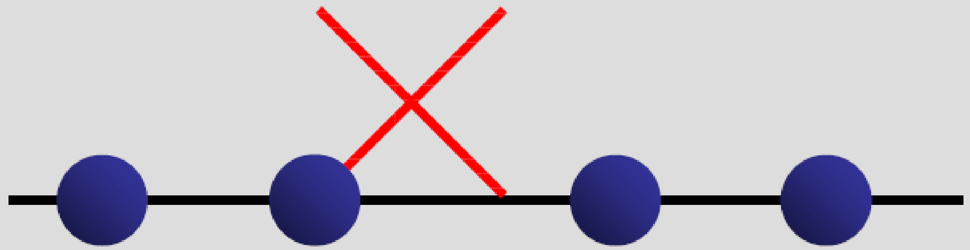
Motivations: Theoretical

A Gas in One Dimension

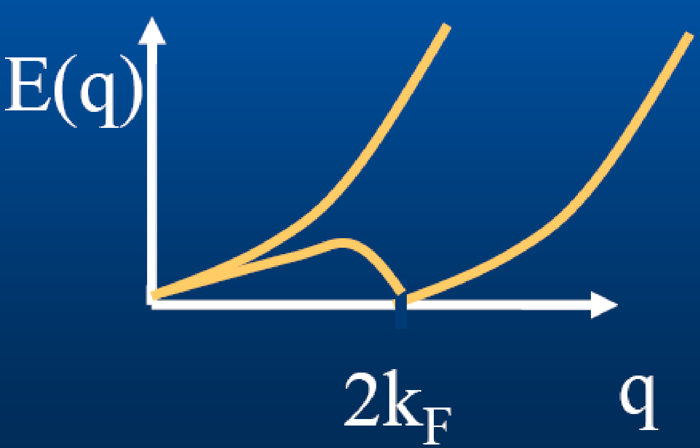


- Increased quantum fluctuations of the phase
- In the homogeneous system: no BEC
- Interactions more important

Motivations: Theoretical

What is special in d=1 ?

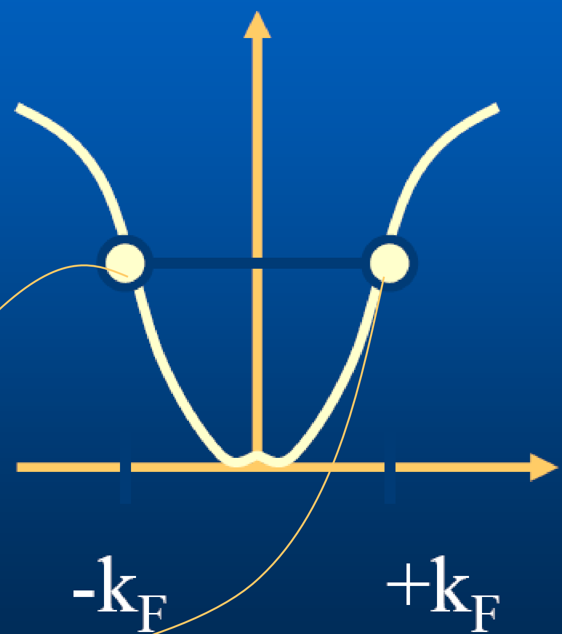
- No individual excitation can exist (only collective ones)



D=1:

Well defined excitations

$$E(q) = v_F q$$



Ψ_L, Ψ_R

BOSONIZATION: SPINLESS INERACTING FERMIONS

THE KINETIC ENERGY

$$\nabla\Phi(x) = -\pi[\rho_R(x) + \rho_L(x)]$$

$$\nabla\Theta(x) = \pi[\rho_R(x) - \rho_L(x)] = \pi\Pi(x)$$

$$H = \int \frac{dx}{2\pi} v_F [(\pi\Pi(x))^2 + (\nabla\Phi(x))^2]$$

- ``Phonon'' Hamiltonian (sound waves of charge)

INTERACTIONS AND LUTTINGER LIQUID MODEL

$$\rho(x)\rho(x') \approx (\nabla\Phi(x))^2$$

$$H = \int \frac{dx}{2\pi} [uK(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\Phi(x))^2]$$

K< 1 : repulsive
K>1 : attractive

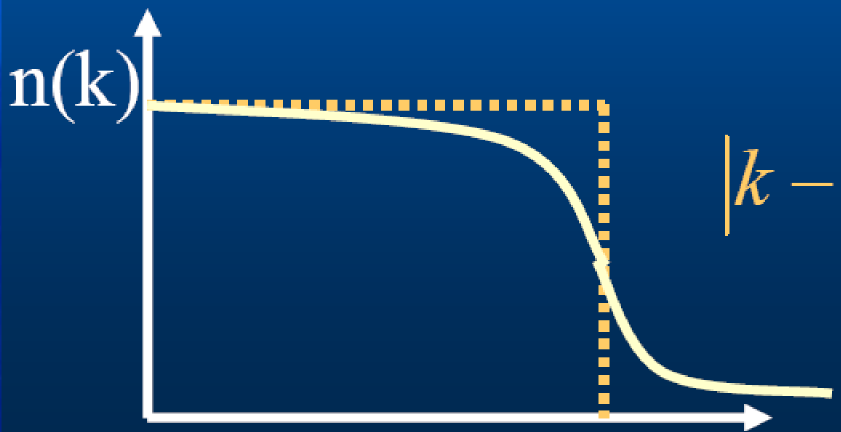
- u velocity of sound

Correlation functions

$$\langle \rho(x)\rho(0)\rangle = \frac{1}{x^2} + \cos(2k_F x)\left(\frac{1}{x}\right)^{2K}$$

$$\langle \psi_R(x)\psi_R^*(0)\rangle = \left(\frac{1}{x}\right)^{\frac{1}{2}[K+K^{-1}]} e^{iArg(\tau/x)}$$

$$K=1 \qquad \langle \psi_R(x)\psi_R^*(0)\rangle = \frac{1}{x - v_F\tau}$$

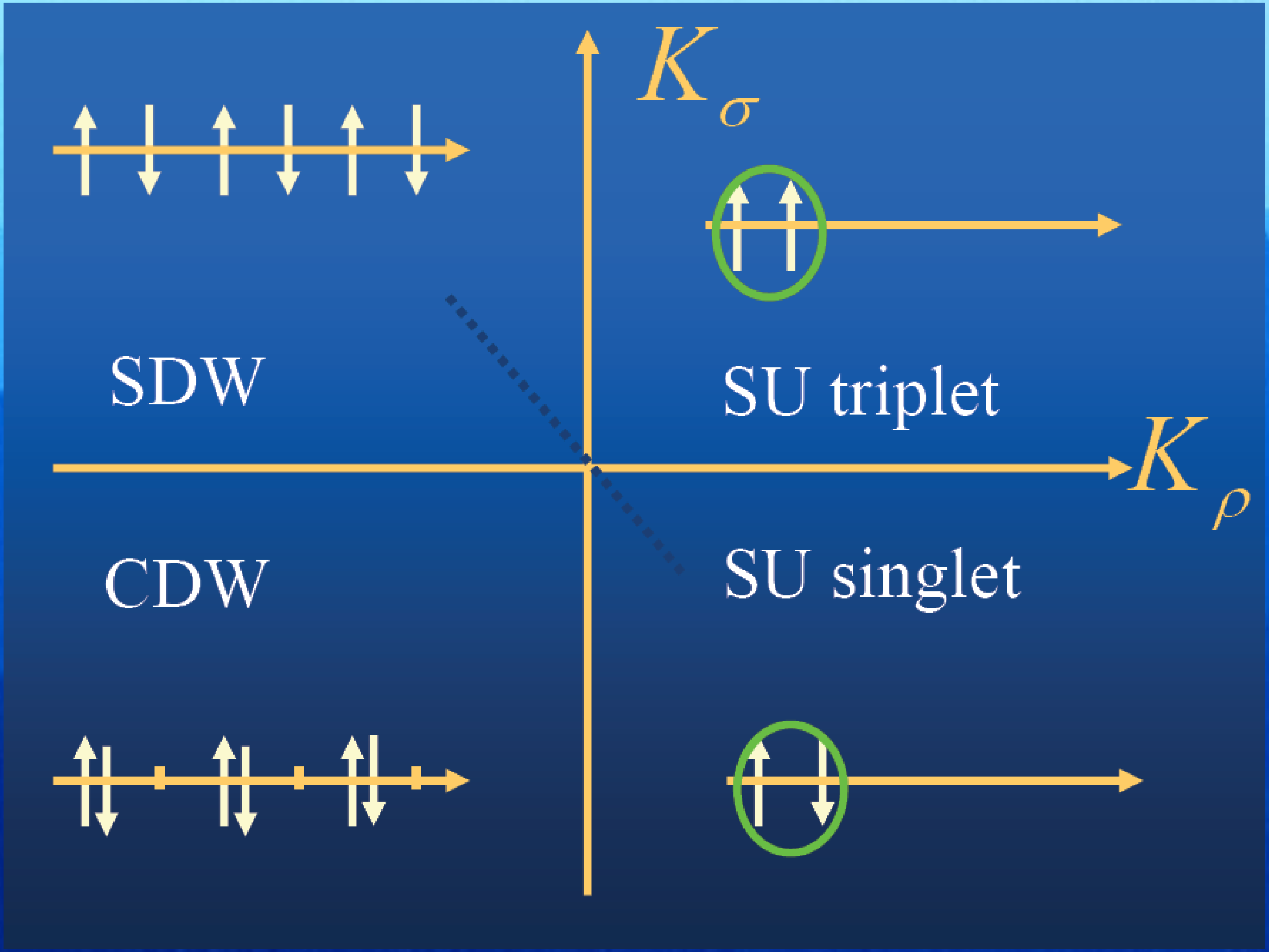


$$|k - k_F|^{\frac{1}{2}[K+K^{-1}]-1}$$

No Landau Quasiparticles !

PHASE DIAGRAM OF 1D FERMIONS WITH SPIN

SPIN-CHARGE SEPARATION



Interacting bosons in one-dimension and Luttinger liquids

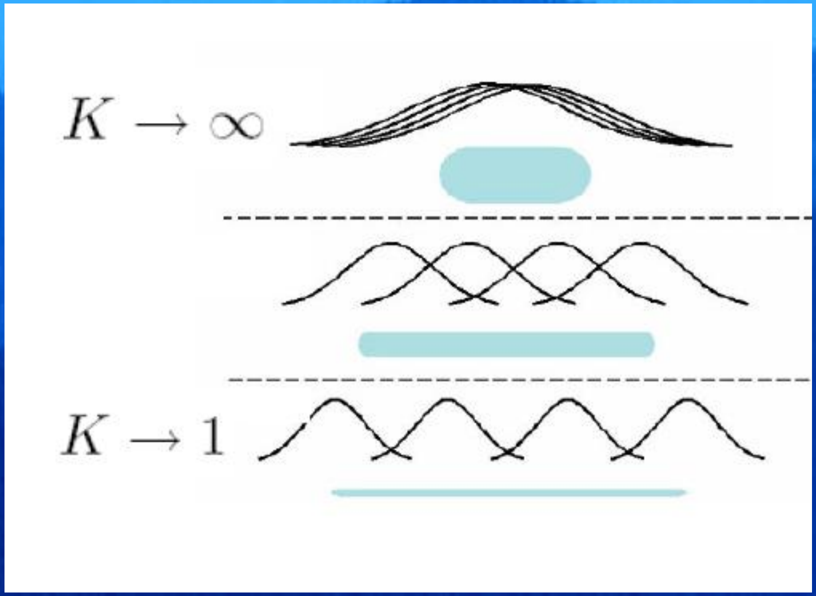
$$H = \int \frac{dx}{2\pi}[uK(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\Phi(x))^2]$$

- u velocity of sound

$$\psi_B^\dagger(x) = [\rho_0 - \frac{1}{\pi}\nabla\phi(x)]^{1/2}\sum_p e^{i2p(\pi\rho_0 x - \phi(x))}e^{-i\theta(x)}$$

$\theta = \pi \int^x dx'\Pi(x')$ = Superfluid phase

$\partial_x\phi$ = density (dual variable)

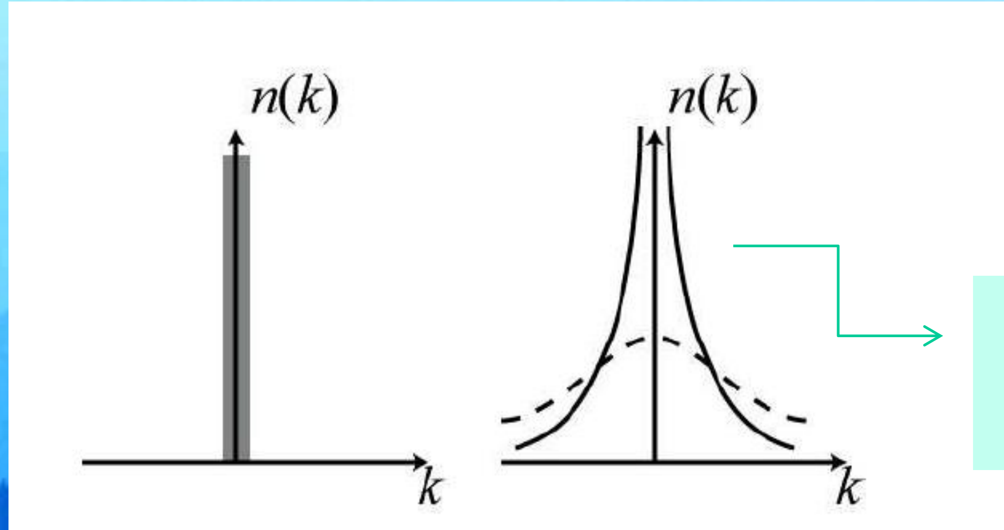


For non-interacting bosons $K = \infty$

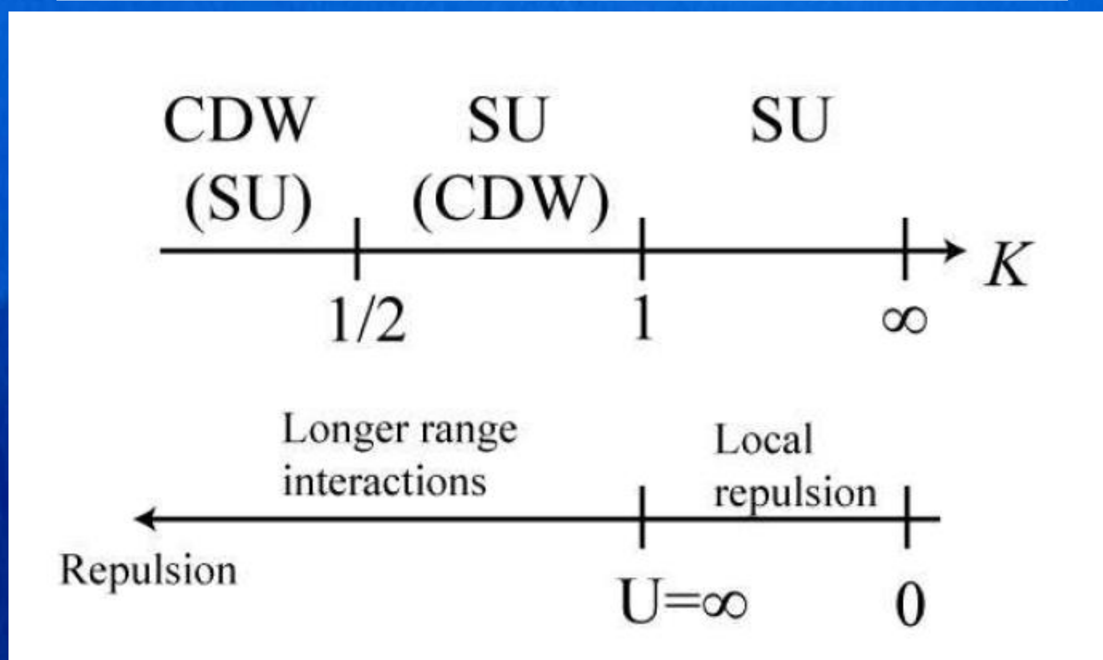
For impenetrable bosons $K = 1$

Luther-Peschel (1975)
Haldane (1980)

PHASE DIAGRAM (BOSONS)



Effect of interactions: power-law divergence with exponent 1-1/2K



Bosonization approach Interaction with a single impurity

$$H_{imp} = \int dx\, V(x)\rho(x)$$

$$= \int dx\, V(x)[-\frac{1}{\pi}\nabla\phi(x) + \rho_0(e^{i(2\pi\rho_0 x - 2\phi(x))} + \text{h.c.})]$$

$$H_f = -\int dx\, V(x)\frac{1}{\pi}\nabla\phi(x)$$

$$\tilde{\phi}(x) = \phi(x) + \frac{K}{u}\int_0^x dy V(y)$$

Effective potential: Backscattering!

$$\int dx\, V(x)\rho_0(e^{i(2\pi\rho_0 x - 2\phi(x))} + \text{h.c.})]$$