Corso di Interazioni Fondamentali
Prova scritta

Available time for the assignment: 1h00m.
Please answer schematically, but completely, with clear handwriting.
Please write first name, family name and "matricola" number.

Appello del 10/1/2022

Problem 1

Consider the fixed target reaction $K^- p \to \pi^0 \Lambda^0$. Determine, also numerically, the following quantities:

1. the minimum energy of the K^- beam for which the reaction can occur;
2. the minimum energy of the K^- beam for which the Λ^0 can be produced at rest in the laboratory reference frame.

SOLUTION:

1. *Since $m_K + m_p > m_\Lambda + m_\pi$ the reaction can happen even with the K at rest. The reaction threshold calculation would yield*

$$E_K = \frac{(m_\Lambda + m_\pi)^2 - m_K^2 - m_p^2}{2m_p} = 240.5 MeV < m_K \tag{1}$$

confirming the observation.

2. *From the conservation of 4-momentum $p_K + p_p = p_\Lambda + p_\pi$ and considering at rest the proton and the Λ one obtains*

$$m_\pi^2 = m_p^2 + m_\Lambda^2 + m_K^2 + 2m_p E_K - 2m_p m_\Lambda - 2m_\Lambda E_K \tag{2}$$

$$E_K = \frac{(m_\Lambda - m_p)^2 + m_K^2 - m_\pi^2}{2(m_\Lambda - m_\pi)} = 722.5 MeV \tag{3}$$

Note that this is kinematically equivalent to the decay $\Lambda \to p K^- \pi^0$ with the proton at rest.

Problem 2

1. Show that neglecting phase space the branching ratios of the decay of τ leptons into electrons, muons, and hadrons are in the approximate ratio 1:1:3.

2. Estimate the fraction of hadronic decays that contain at least one kaon.

SOLUTION

- *At the lowest order τ decau proceeds through the emission of a virtul W boson which successfully the following processes (assuming a τ^-):*

$$W^- \to e^- \bar{\nu}_e \quad W^- \to \mu^- \bar{\nu}_\mu \quad W^- \to d\bar{u} \quad W^- \to s\bar{u} \tag{4}$$

  with coupling 1, 1, $\cos\theta_c$ and $\sin\theta_c$ respectively (multiplied by g_W). Higher mass quarks are not kinematically allowed. The relative branching fractions in electrons, muons, are therefore immmediately 1:1. The branching fraction in hadrons relative to electrons is $N_{colors}(\cos^2\theta_c + \sin^2\theta_c) = 3$. This would imply $BR(\tau \to e^- \bar{\nu}_e \nu_\tau)$=20%. The experimental number is slightly lower (17%) because of QCD corrections.

- *The ratio of strange hadronic decays to total hadronic decays at first order is* $\sin^2\theta_c/(\cos^2\theta_c + \sin^2\theta_c) = \sin^2\theta_c = 4.8\%$

Problem 3

Proton-antiproton annihilations *at rest* proceed via the formation of a $\bar{p}p$ hydrogen-like atom called *protonium*, whose initial state has well defined quantum numbers.

1. For every initial state in the following table, compute the J^{PC} quantum numbers and indicate wether the indicated decays can occur. In case they are forbidden, indicate the reason (e.g. parity, C-parity, ... conservation).

		J^{PC}	$\pi^+\pi^-$	$\pi^0\pi^0$	K^+K^-
S wave	$^1S_0$				
	$^3S_1$				
P wave	$^1P_1$				
	$^3P_0$				
	$^3P_1$				
	$^3P_2$				

2. Express the probabilities of the two processes involving pions in terms of isospin amplitudes A_0, A_1, A_2, assuming that the protonium has an isospin wavefunction:

$$\Psi(\bar{p}p) = \frac{1}{\sqrt{2}} \left(|1,0\rangle - |0,0\rangle \right). \tag{5}$$

3. In case of $\bar{p}p \to \pi^+\pi^-, \pi^0\pi^0$ annihilation from a $^3P_0$ state, what are the relevant isospin amplitudes? What is the relative branching ratio of the two channels?

SOLUTION

1. *The spectroscopic notation indicates the $\bar{p}p$ orbital angular momentum L with a letter (0=S, 1=P, ...), the other two numbers being the spin multiplicity and the total angular momentum J: $^{2S+1}L_J$. J is therefore readily written by copying the J of the spectroscopic notation. For parity and $C-$parity we must take into account the difference between fermion-antifermion and boson-antiboson pairs.*

   For a fermion-antifermion system (such as $\bar{p}p$) we have that $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$, while for spinless boson-antibosons (such as πs and Ks) $P = C = (-1)^L$, L being their orbital *angular momentum. Note that $\pi^0\pi^0$ case has the further constraint given by Bose-Einstein statistics that requires its wavefunction to be even under particle exchange forcing $L =$even $\Rightarrow P = C = +$.*

   For angular momentum conservation, the orbital angular momentum of $\pi\pi$ and K^+K^- pairs are equal to the total *angular momentum of the $\bar{p}p$ pair, namely J.*

   Applying these rules we obtain the results in the following table. For instance the parity of the $\pi^+\pi^-$ pair is $(-1)^J$ hence the state is not reachable from $^1S_0$.

   Note that the K^+K^- case is identical to $\pi^+\pi^-$, so the last column is just a copy of the charged pions one.

		J^{PC}	$\pi^+\pi^-$	$\pi^0\pi^0$	K^+K^-
S wave	$^1S_0$	0^{-+}	*No (P)*	*No (P)*	*No (P)*
	$^3S_1$	1^{--}	*Allowed*	*No (C)*	*Allowed*
P wave	$^1P_1$	1^{+-}	*No (P)*	*No (P)*	*No (P)*
	$^3P_0$	0^{++}	*Allowed*	*Allowed*	*Allowed*
	$^3P_1$	1^{++}	*No (C,P)*	*No (C,P)*	*No (C,P)*
	$^3P_2$	2^{++}	*Allowed*	*Allowed*	*Allowed*

2. *In term of isospin the antiproton is a $|^1/_2, -^1/_2\rangle$ hence the initial state is a mixture of $I = 0$ and $I = 1$, as described in the text. From the Clebsch-Gordan coefficient table one gets:*

$$\langle\pi^+\pi^-|\bar{p}p\rangle \propto \frac{1}{2}A_1^2 + \frac{1}{\sqrt{6}}A_0^2 \quad (6)$$

$$\langle\pi^0\pi^0|\bar{p}p\rangle \propto \frac{1}{\sqrt{6}}A_0^2. \quad (7)$$

3. *For a pion-antipion system C-parity is related to isospin according to $C = (-1)^I$, hence from a $^3P_0$ initial state, $I_{\pi^+\pi^-} = I_{\pi^0\pi^0} = 0$ therefore only A_0 contributes and $BR(\bar{p}p \to \pi^-\pi^+) = 2\times\ BR(\bar{p}p \to \pi^0\pi^0)$ (recall that we must count both $\pi^+\pi^-$ and $\pi^-\pi^+$ pairs which are indistinguishable final states!).*