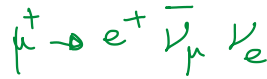
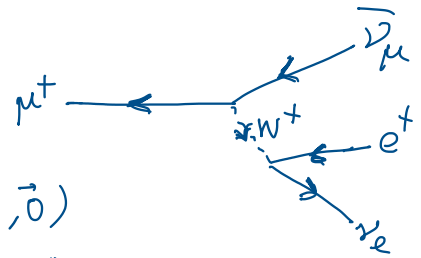


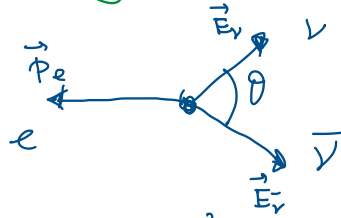
3 Three body decay

1. Muon decay
2. Multibody decay
3. Tritium decay and neutrino mass

• 3 body μ decay



CM of μ



$$P_\mu = (m_\mu, \vec{0})$$

$$P_e = (E_e, \vec{p}_e)$$

$$P_{\nu, \bar{\nu}} = (E_{\nu, \bar{\nu}}, \vec{p}_{\nu, \bar{\nu}})$$

$$P_\mu = P_e + P_{\nu} + P_{\bar{\nu}} \quad (P_\mu - P_e)^2 = (P_{\nu} + P_{\bar{\nu}})^2 \geq 0$$

$$m_\mu^2 + m_e^2 - 2m_\mu E_e = 0 + 0 + 2E_\nu E_{\bar{\nu}} (1 - \cos\theta)$$

$$E_e = \frac{m_\mu^2 + m_e^2 - 2E_\nu E_{\bar{\nu}} (1 - \cos\theta)}{2m_\mu}$$

$$\begin{aligned} & [E_\nu, E_{\bar{\nu}}, \theta_1, \theta_2] \\ & E_\nu \sin\theta_1 = E_{\bar{\nu}} \sin\theta_2 \\ & E_\nu \cos\theta_1 + E_{\bar{\nu}} \cos\theta_2 = E_e \end{aligned}$$

Number of free parameters: 3×3 -momenta = 9

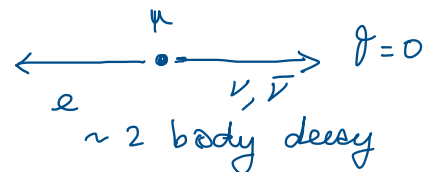
4-momentum conserv. -4

decay plane -2

electron direction -1

$$\frac{-4-2-1}{2} = \theta, E_\nu$$

$$E_{\max} = \frac{m_\mu^2 + m_e^2}{2m_\mu} \sim \frac{m_\mu}{2} = 52.8 \text{ MeV}$$



$$E_{\min} = m_e \sim 0.511 \text{ MeV}$$

$$E_\nu = E_{\bar{\nu}} = \frac{(m_\mu - m_e)}{2}$$

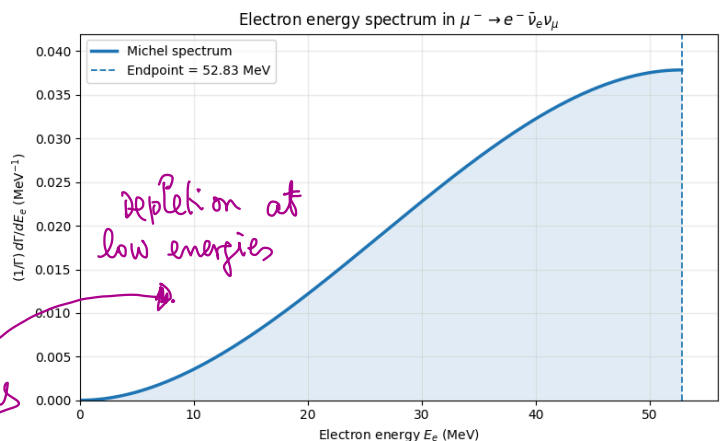


Michel spectrum

$$\frac{1}{\Gamma} \frac{d\Gamma}{dE_e} = \frac{4}{m_\mu} \left(\frac{2E_e}{m_\mu} \right)^2 \left(3 - \frac{4E_e}{m_\mu} \right)$$

• Phase space

• Dynamics of weak interactions (V-A)



N-Body decay

$$M \rightarrow m_1 + m_2 + \dots + m_N \quad E_{1, \max} \text{ min } ?$$

$$\vec{p} = \vec{p}_1 + [\vec{p}_2 + \dots + \vec{p}_N]$$

nel CM $\vec{p} = (M, \vec{0})$

$$M^2 + m_1^2 - 2ME_1 = S_{2:N}$$

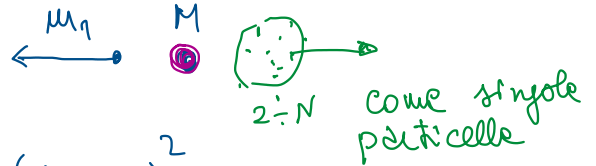
$$E_1 = \frac{M^2 + m_1^2 - S_{2:N}}{2M}$$

$$E_{1, \max} = \frac{M^2 + m_1^2 - \left(\sum_{i=2}^N m_i\right)^2}{2M}$$

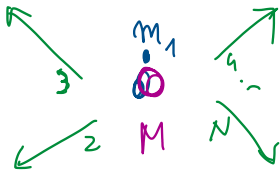
$$(\vec{p} - \vec{p}_1)^2 = \left(\sum_{i=2}^N \vec{p}_i\right)^2 = S_{2:N}$$

$$S_{2:N} = \left[\text{energia disponibile nel CM delle particelle } 2 \dots N \right]^2$$

$$S_{2:N} \geq \left(\sum_{i=2}^N m_i\right)^2$$



$$E_{1, \min} = m_1 \rightarrow S_{2:N} = (M - m_1)^2$$



$$T_{1, \max} = E_{1, \max} - m_1 = \frac{(M - m_1)^2 - \left(\sum_{i=2}^N m_i\right)^2}{2M}$$

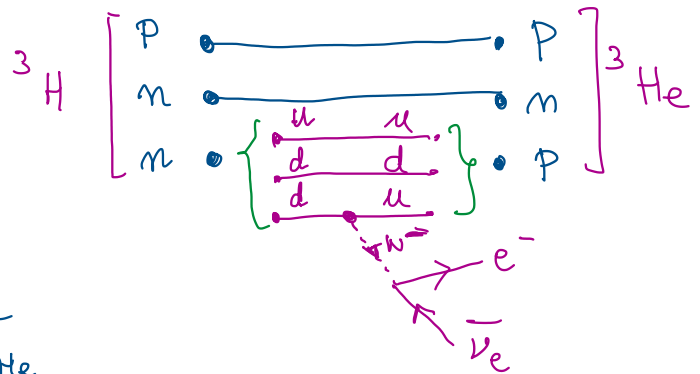
$$T_{1, \min} = 0$$

Tritium decay - Nuclear β decay



$$Q = m_{{}^3\text{H}} - m_{{}^3\text{He}} - m_e \approx 18.59 \text{ keV}$$

$$= T_e + E_\nu + T_{\text{recoil}}$$

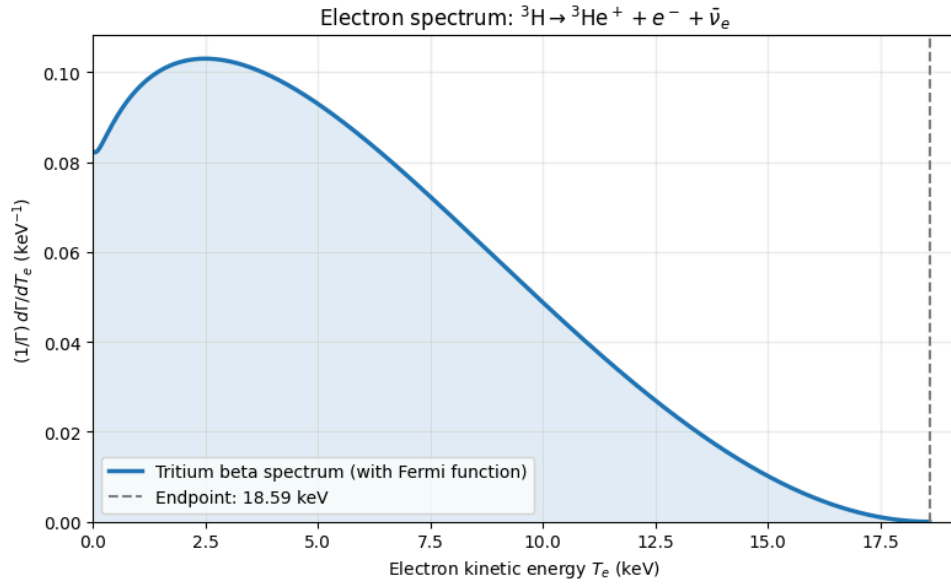
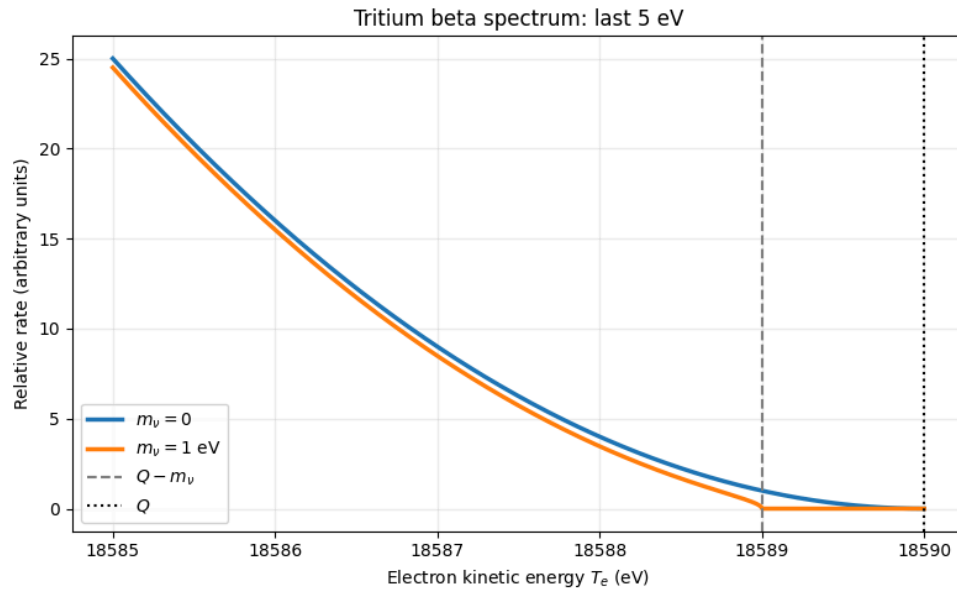


$$T_{e, \max} = \frac{(m_{{}^3\text{H}} - m_e)^2 - m_{{}^3\text{He}}^2}{2m_{{}^3\text{H}}} =$$

$$\approx (m_{{}^3\text{H}} - m_e - m_{{}^3\text{He}}) \left[\frac{m_{{}^3\text{H}} - m_e + m_{{}^3\text{He}}}{2m_{{}^3\text{H}}} \right] \approx Q$$

For massive neutrino

$$T_{e, \max} = \frac{(m_{{}^3\text{H}} - m_e)^2 - (m_{{}^3\text{He}} + m_\nu)^2}{2m_{{}^3\text{H}}} \approx Q - m_\nu$$



Katrin Experiment - <https://www.katrin.kit.edu/1271.php>
<https://www.science.org/doi/10.1126/science.adq9592>
<https://arxiv.org/abs/2406.13516>

Latest result (2025): $m(\nu) < 0.45 \text{ eV}$

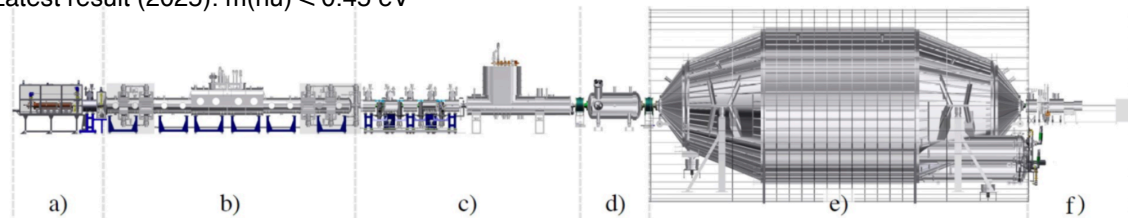


FIG. 1. The major components of the KATRIN beam line consist of a) the Rear Section for diagnostics, b) the windowless gaseous tritium source WGTS, c) the pumping section with the DPS and CPS cryostats, and a tandem set-up of two MAC-E-filters: d) the smaller pre-spectrometer and e) the larger main spectrometer with its surrounding aircoil system. This system transmits only the highest-energy β -decay electrons onto f) the solid-state detector where they are counted.

$$T_2 \rightarrow {}^3\text{He}T^+ + e^- + \bar{\nu}_e.$$

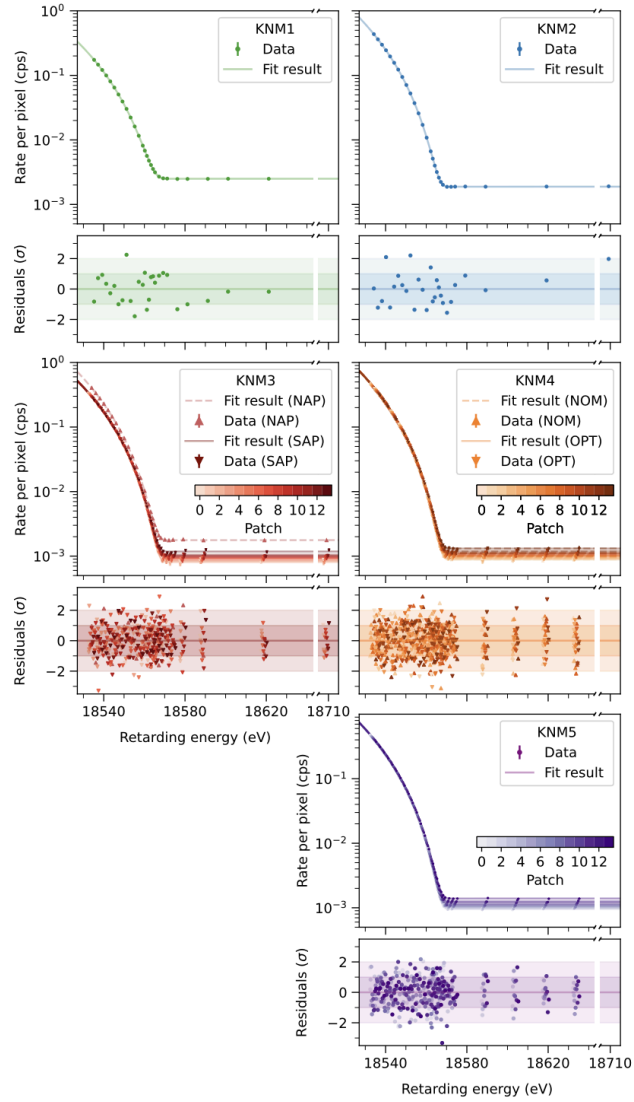


Figure 2: Spectra, fit models and normalized residuals of each measurement campaign. The KNM3-SAP, KNM4-NOM, and KNM4-OPT, and KNM5 data are subdivided into 14 detector patches. The squared neutrino mass is a common fit parameter over all data sets.