

2 Reaction threshold

1. Threshold with target in Lab System
2. Threshold with final particle in Lab System
3. Threshold with photon

Threshold : minimum (kinetic) energy of the impinging particle for which the reaction is allow kinematically

$$p_1 + p_2 \rightarrow \text{final state} : (p_1 + p_2)^2 = s \geq \underline{\text{min}} s \text{ of final state}$$

For a multiparticle system

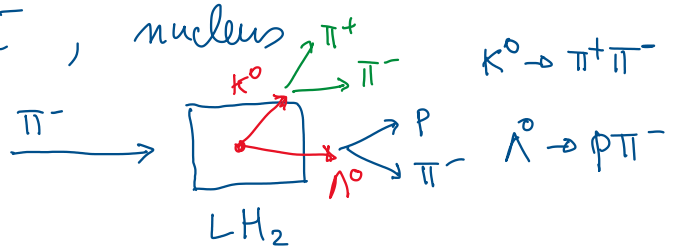
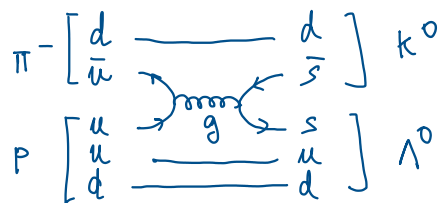
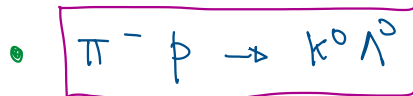
$$s = (\sum p_i)^2 = (\sum E_i^*)^2 = \sum (m_i + T_i^*)^2 \geq \sum (m_i)^2$$

in CM $\sum \vec{p}_i = 0$ $\uparrow$ kin energy in CM

The min s of final state is $\boxed{s_{\min} = \sum_f (m_f)^2}$ (threshold)

① Fixed target experiments

target can be $p, n, e^-, \text{ nucleus}$



$$(p_\pi + p_p)^2 = (p_K + p_\Lambda)^2 \quad \text{INV}$$

$\downarrow$ LAB $\downarrow$ CM

$$m_\pi^2 + m_p^2 + 2 \boxed{E_\pi} m_p = (E_K^* + E_\Lambda^*)^2$$

$$E_\pi = \frac{(E_K^* + E_\Lambda^*)^2 - m_\pi^2 - m_p^2}{2m_p}$$

At threshold $E_K^* = m_K, E_\Lambda^* = m_\Lambda$

$$E_{\pi, \text{th}} = \frac{(m_K + m_\Lambda)^2 - m_\pi^2 - m_p^2}{2m_p}$$

LAB $\begin{cases} p_\pi = (E_\pi, \vec{p}_\pi) \\ p_p = (m_p, \vec{0}) \end{cases}$

CM $\begin{cases} p_K^* = (E_K^*, \vec{p}^*) \\ p_\Lambda^* = (E_\Lambda^*, -\vec{p}^*) \\ \vec{p}_K^* + \vec{p}_\Lambda^* = \vec{p}^* - \vec{p}^* = 0 \end{cases}$

$$\begin{aligned} T_{\pi, \text{th}} &= E_{\pi, \text{th}} - m_\pi = \\ &= \frac{(m_K + m_\Lambda)^2 - (m_\pi + m_p)^2}{2m_p} \end{aligned}$$

$$m_\pi \sim 140 \text{ MeV}$$

$$m_p \sim 938 \text{ MeV}$$

$$m_K \sim 500 \text{ MeV}$$

$$m_\Lambda \sim 1116 \text{ MeV}$$

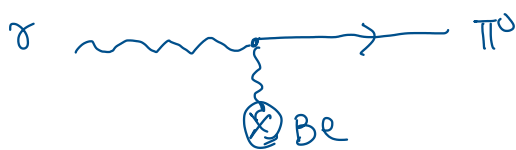
$$T_\pi = \frac{(500+1116)^2 - (140+938)^2}{2 \times 938} = 772 \text{ MeV}$$

$$E_\pi = T_\pi + m_\pi = 772 + 140 = 912 \text{ MeV}$$

NB: final particles are at rest in the CM NOT in the LAB
they move in the LAB as a single particle of mass $M_f = \sum m_f$

Photoproduction of π^0

$$\gamma \text{ Be} \rightarrow \pi^0 \text{ Be}$$



$$p_\gamma = (E_\gamma, E_\gamma \hat{z})$$

$$p_{\text{Be}} = (m_{\text{Be}}, 0)$$

$$0 + m_{\text{Be}}^2 + 2m_{\text{Be}} E_\gamma = (m_\pi + m_{\text{Be}})^2$$

$$E_\gamma = \frac{(m_\pi + m_{\text{Be}})^2 - m_{\text{Be}}^2}{2m_{\text{Be}}} = \frac{m_\pi^2 + 2m_\pi m_{\text{Be}} + \cancel{m_{\text{Be}}^2} - \cancel{m_{\text{Be}}^2}}{2m_{\text{Be}}} = m_\pi + \frac{m_\pi^2}{2m_{\text{Be}}}$$

$$\frac{m_\pi^2}{2m_{\text{Be}}} = \frac{(0.135)^2}{2 \times 8} \approx 1.13 \text{ MeV}$$

$$\text{on Hydrogen } \frac{m_\pi^2}{2m_p} \approx 9 \text{ MeV}$$

$$\pi^0 \rightarrow \gamma\gamma \quad \tau \sim 10^{-17} \text{ s}$$

$$m_{\pi^0} = 135 \text{ MeV} \quad m_{\text{Be}} \sim 8 \text{ GeV}$$

t-channel

⊗ can take a lot of momentum without changing energy much
→ Brick wall

the heavier the nucleus,
the smaller.

For $m_{\text{target}} \gg m_p \rightarrow E_\gamma \approx m_{\pi^0}$

Final state at rest - exam 21/9/2023

$$K^- p \rightarrow \pi^0 \Lambda \quad (1) \text{ threshold}$$

(2) Min E_K to have Λ at rest in LAB

$$p_K = (E_K, \vec{p}_K) \quad p_p = (m_p, 0)$$

$$p_{\pi^0} = (E_\pi, \vec{p}_\pi) \quad p_\Lambda = (m_\Lambda, 0)$$

$$m_\pi^2 = m_K^2 + m_p^2 + m_\Lambda^2 + 2m_p E_K = 2m_p m_\Lambda - 2E_K m_\Lambda$$

$$E_K = \frac{(m_\Lambda - m_p)^2 + m_K^2 - m_\pi^2}{2(m_\Lambda - m_p)} = 722.5 \text{ MeV}$$

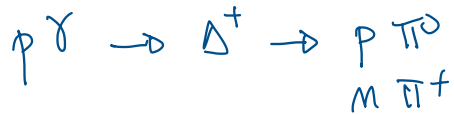
(same as $\Lambda \rightarrow K p \pi^0$ with proton at rest)

$$E_K = \frac{(m_{\pi^0} + m_\Lambda)^2 - m_p^2 - m_K^2}{2m_K} = 240 \text{ MeV}$$

$m_K + m_p > m_{\pi^0} + m_\Lambda$
 500 938 135 1116

$< m_K$

3. Scattering of photons of Cosmic Microwave Background

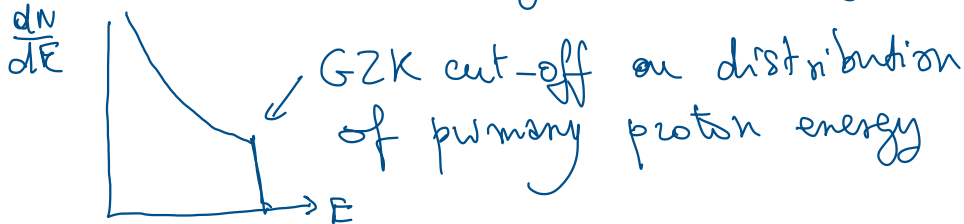


$$T \sim 3 K$$

$$E_\gamma = 25 \text{ meV} \times \frac{3 K}{300 K} = 2.5 \times 10^{-4} \text{ eV}$$

1966 : Greisen - Zetsepin - Kuzmin (GZK)

Ultra High Energy protons above E_{GZK} necessary to produce a π will be absorbed by CMB scattering $\rightarrow$ cut-off



E_{GZK} ? Cannot use the method above for 0-mass target
Target (γ) cannot be at rest!

$$s = (p_p + p_\gamma)^2 = m_p^2 + 0 + 2 E_p E_\gamma - 2 \vec{p}_p \cdot \vec{p}_\gamma = 2 |\vec{p}_p| E_\gamma \cos \theta$$

For a given E_p s is max for head-on collisions $\cos \theta = -1$

$$\vec{p}_p \quad \leftarrow \quad \vec{p}_\gamma$$

$$m_p^2 + 2(\vec{p}_p + \vec{p}_\gamma) E_\gamma \geq (m_p + m_{\pi^0})^2$$

For UHE protons $|\vec{p}_p| \sim E_p$ $E_p \geq \frac{(m_p + m_{\pi^0})^2 - m_p^2}{4 E_\gamma} =$

$$E_{GZK} = \frac{2m_p m_{\pi^0} + m_{\pi^0}^2}{4 E_\gamma} = \frac{1}{2} \frac{m_p m_{\pi^0}}{E_\gamma} \left[1 + \frac{1}{2} \frac{m_{\pi^0}}{m_p} \right] =$$

$$= \frac{0.5 \times 1 \times 0.14}{2.5 \times 10^{-13}} [1 + 0.07] \text{ GeV} \approx \boxed{0.3 \times 10^{12} \text{ GeV}}$$

Real number must take into account Boltzmann distribution

• In the proton rest system

$$s = m_p^2 + 2 E_\gamma^* m_p = (m_p + m_{\pi^0})^2$$

$$E_\gamma^* = \frac{\cancel{m_p^2} + 2m_p m_{\pi^0} + m_{\pi^0}^2 - \cancel{m_p^2}}{2m_p} = m_{\pi^0} \left(1 + \frac{1}{2} \frac{m_{\pi^0}}{m_p} \right)$$

$$\sim 135 \text{ MeV}$$

$\rightarrow$ in the p rest system the γ has enough energy to produce a π^0 !