4 Dalitz Plot

Resonance: lifetime is so short ($\tau \sim 10^{-23}$ s) that mass is uncertain

$$\sigma(i \to R \to f) = \frac{4\pi}{k_a^{*2}} \frac{(2J+1)}{(2S_a+1)(2S_b+1)} \frac{\Gamma_i \Gamma_f / 4}{(E-M_R)^2 + \Gamma^2/4}$$

spin of resonance; initial/final widths

$(a+b) \to R \to f$

k_a^*: CM momentum; S_a, S_b: initial particles spin; M_R: Resonance mass; Γ: total width

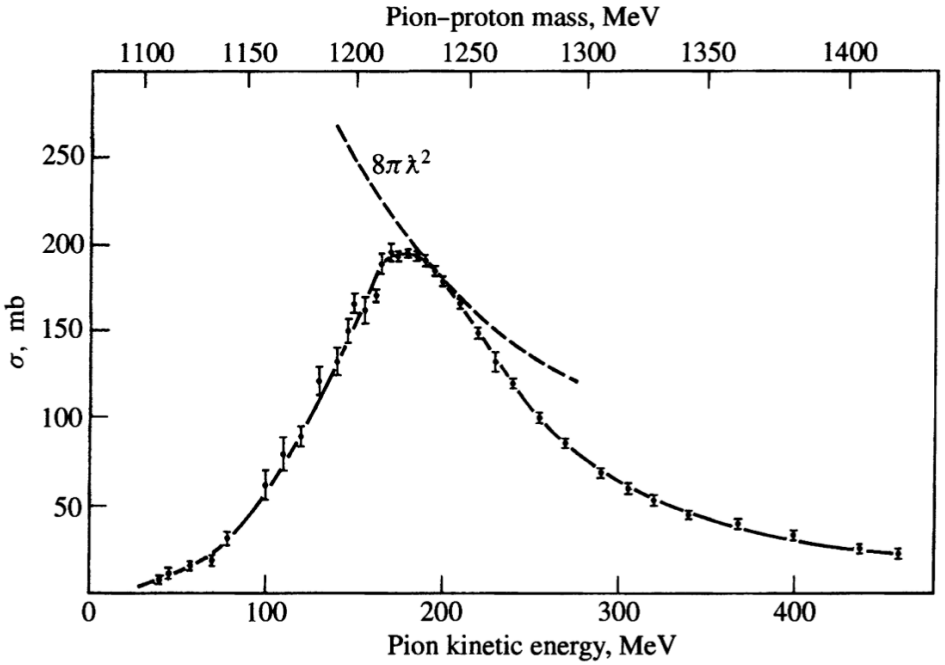
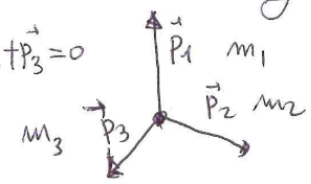


Fig. 2.11. The $\pi^+ p$ elastic scattering cross-section in the region of the $\Delta^{++}(1232)$ resonance. The central mass is 1232 MeV and the width is $\Gamma = 120$ MeV. Note that the formula (2.28) holds strictly for a narrow resonance. For a broad resonance with width comparable with the central mass, such as the Δ^{++}, the final-state phase-space factor varies appreciably over the width so that, in comparison with Figure 2.10, the resonance curve appears asymmetric.

2 Dalitz plot and three body phase space Bellini

Consider a 3-body final state in the CM

$\vec{p}_1+\vec{p}_2+\vec{p}_3=0$



Invariant masses $m_{23}^2=(p_2+p_3)^2$

$T_i=E_i-m_i$ $\quad m_{13}^2=(p_1+p_3)^2$

It can be demonstrated that the phase is proportional to

$$\int dE_1\,dE_2 \;\propto\; \int dm_{23}^2\,dm_{13}^2 \;\propto\; \int dT_1\,dT_1$$

energy — inv mass squared — kin energy.

Since $E_i=m_i+T_i \rightarrow dE_i=dT_i$

Since $P=p_1+p_2+p_3=(\sqrt{s},\vec{0}) \rightarrow m_{23}^2=(p_2+p_3)^2=(P-p_1)^2$

$m_{23}^2=s+m_1^2-2\sqrt{s}E_1 \rightarrow dm_{23}^2=(-2\sqrt{s})\,dE_1$

$\rightarrow$ the three forms are equivalent. (3-momenta)

Dim. $R_3 \propto \int \frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\frac{d^3p_3}{2E_3}(2\pi)^4\delta(E_1+E_2+E_3-\sqrt{s})\,\delta^3(\vec{p}_1+\vec{p}_2+\vec{p}_3)$

Integrate over $\vec{p}_3$, remove factors:

$$R_3 \propto \int \frac{d^3p_1\,d^3p_2}{E_1E_2E_3}\delta(E_1+E_2+E_3-\sqrt{s}) = \int \frac{p_1^2\,dp_1\,d\Omega_1\,p_2^2\,dp_2\,d\Omega_2}{E_1E_2E_3}\cdot(4\pi)$$

Fix the $\vartheta_{12}=\vartheta_2-\vartheta_1$ angle and integrate on $\vartheta_1,\varphi_1,\varphi_2$

$$R_3 \propto \int \frac{p_1^2\,dp_1\,p_2^2\,dp_2\,(4\pi)\cdot(2\pi)\,d\cos\vartheta_{12}}{E_1E_2E_3}\,\delta(\;)$$

Using $\vec{p}_3 = -(\vec{p}_1 + \vec{p}_2)$: $\vec{p}_3^{\,2} = \vec{p}_1^{\,2} + p_2^2 + 2p_1 p_2 \cos\theta_{12}$

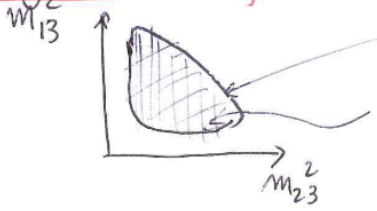
For constant p_1, p_2 : $p_3 dp_3 = p_1 p_2 d\cos\theta_{12}$

Also, $E_i^2 = m_i^2 + \vec{p}_i^{\,2}$ $\rightarrow$ $2E_i dE_i = 2p_i dp_i$ (3-momenta)

$$R_3 \propto \int \frac{p_1 dp_1 \cdot p_2 dp_2 \cdot p_3 dp_3}{E_1 E_2 E_3} \delta(E_1 + E_2 + E_3 - \sqrt{s}) =$$

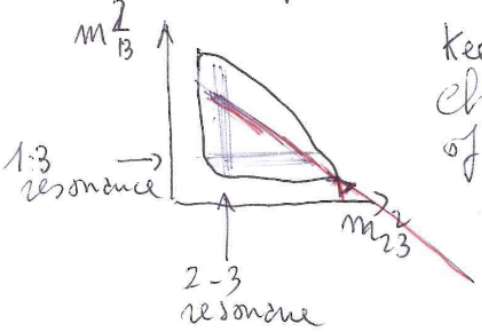
$$= \int dE_1 dE_2 dE_3 \cdot \delta(\ ) = \int dE_1 dE_2 \quad \text{(integrating on } E_3\text{)}$$

$\Rightarrow$ Uniform distribution in $E_1 E_2$, $m_{23}^2 m_{13}^2$, $T_1 T_2$ planes. **Any deviation from uniformity is dynamics.**



Kinematically allowed area uniform distribution in absence of dynamical features.

Resonances appear as denser bands



Keep in mind there may be multiple charge states or multiple possibility of selecting particle 1 2 3.

$$m_{13}^2 + m_{23}^2 + m_{12}^2 = (P - p_2)^2 +$$

$$+ (P - p_1)^2 + (P - p_3)^2 =$$

$$= 3P^2 + m_1^2 + m_2^2 + m_3^2 - 2P(p_1 + p_2 + p_3) = 5 + m_1^2 + m_2^2 + m_3^2$$

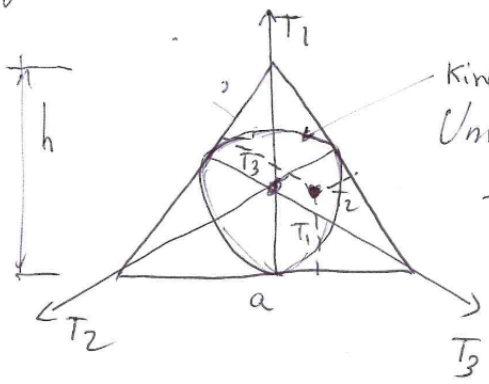
$p_1 + p_2 + p_3 = P$

The flat phase space distribution can also be affected by isospin/spin/parity of the final state. Example 3pi system detailed in Bettini.

3-π system

Dalitz construction for 3π system (K decay only)

Use equilateral triangle to represent the 3π kinetic energies T_1, T_2, T_3. $T_1+T_2+T_3 = E_1+E_2+E_3 - 3m_\pi$ = constant



Kinematical limit → Circle for m=0

Uniform phase space

$T_1+T_2+T_3$ = const because of triangle properties

Dim: Area $= \frac{1}{2}ah = \frac{1}{2}[aT_1 + aT_2 + aT_3] = \frac{1}{2}a[T_1+T_2+T_3]$.

Assign of angular momentum states lead at different densities in the Dalitz plot.

Spin-parity-isospin of 3π system

K, η, ω decay in 3π. Depending on I, J, P the Dalitz plot distribution are different → identify state

Note that this is not necessarily the same of the decaying meson since I is not conserved in EM & Weak interactions

We go in the CM frame $\vec{p}_1+\vec{p}_2+\vec{p}_3=0$

Look at combinations with at least 2 charged π

$\pi^+\pi^-\pi^+$, $\pi^+\pi^-\pi^-$, $\pi^+\pi^-\pi^0$ $\quad Q=-1,0,+1$

I can be in general 0, 1, 2, 3, but only $I=2,3$ mesons have mesons have not been observed (impossible with $q\bar{q}'$ state) → only $I=0$, 1.

Method • call the three π's 1, 2, 3
• Combine 12 and then add 3

Spin Spin of π is 0, only angular momentum exists

$\vec{I}=(\vec{I}_1+\vec{I}_2)+\vec{I}_3=\vec{I}_{12}+\vec{I}_3$ $\qquad \vec{J}=(\vec{J}_1+\vec{J}_2)+\vec{J}_3=\vec{L}_{12}+\vec{L}_3$.

J^P combinations $\qquad P=(-1)^3(-1)^{\ell_{12}}(-1)^{\ell_3}=(-1)^{\ell_{12}+\ell_3+1}$

(intrinsic)

If $J=0$ → $\ell_{12}=\ell_3$ → $P=-1$ → 0^+ cannot exist
Since $P_{intrinsic}$ is -1, the space part transforms as $J^{(-P)}$

→ Now construct the space part of the amplitude (remember $\psi=\psi_{space}\cdot\psi_{isospin}$) using the available Kinematical quantities: (Vector, Pseudo, Axial vector)

V: $\vec{p}_1,\vec{p}_2,\vec{p}_3$ $\left(\sum\vec{p}_i=0\right)$; S: E_1,E_2,E_3 $\left(\sum E_i=\sqrt{s}\right)$

AV: $\vec{q}=\vec{p}_1\times\vec{p}_2=\vec{p}_2\times\vec{p}_3=\vec{p}_3\times\vec{p}_1$

The full $\mathcal{M}$ must be symmetric under exchange (Bosons)

$I=0$ $\qquad \pi^+\pi^-\pi^0$

Since $I(\pi_3)=1$, $I=0 \Rightarrow I_{12}=1$

In $1\otimes1=0\oplus1\oplus2$ $\quad$ 2 is symmetric $(|+1\rangle|+1\rangle)$
1 is antisymmetric $(|+1\rangle|0\rangle-|0\rangle|+1\rangle)$

Since this is true for any choice of π's, it means that the isospin part is completely antisymmetric. AS

This implies that the ψ_{space} must be completely AS under particle exchange.

can be $\pi^+\pi^+\pi^0$ or $\pi^+\pi^+\pi^-$ / $\pi^+\pi^-\pi^-$

• $\pi^+\pi^+\pi^-$ or $\pi^-\pi^-\pi^+$ $\quad$ π_3 is the different one
1 2 3 $\qquad$ 1 2 3

$I_{12}=2 \Rightarrow \psi_{isos}$ Sym $(1\leftrightarrow2)$ → also ψ_{space} Sym $(1\leftrightarrow2)$.

• $\pi^+\pi^-\pi^0$
1 2 3

$I_{12}=1$ NO, because $1\otimes1$ $\qquad I_{12}\ \ I_3$
$|1,0\rangle$ does not contain $|1,0\rangle|1,0\rangle$
$|1,0\rangle=\frac{1}{\sqrt{2}}(|1\rangle|-1\rangle-|-1\rangle|1\rangle)$

$I_{12}=0$ or $I_{12}=2$: Symmetric under $1\leftrightarrow2$

Conclusion: need symmetric under $1\leftrightarrow2$

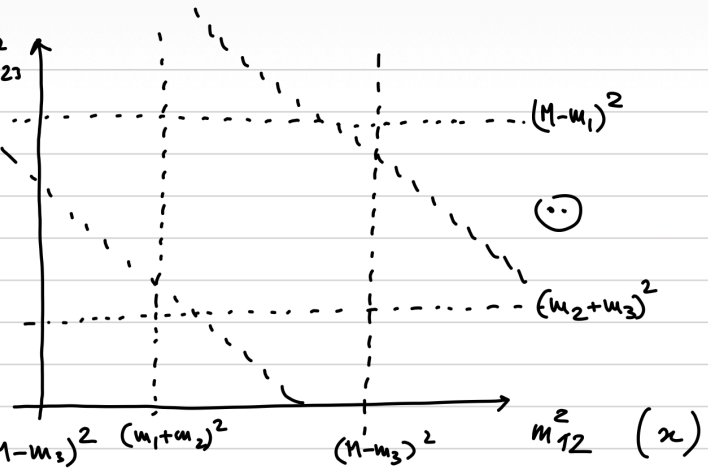
From the distribution in the Dalitz plot can extract the quantum numbers of the 3pi final state

$K\bar{p} \to K^+ K^- \Lambda$
1 2 3

limiti?

$m_{12}^2 = M^2 + m_3^2 - 2ME_3$

E_3 : min m_3 ; max $\dfrac{M^2+m_3^2-(m_1+m_2)^2}{2M}$



m_{12}^2 : max $M^2+m_3^2-2Mm_3 = (M-m_3)^2$; min $M^2+m_3^2-\left[M^2+m_3^2-(m_1+m_2)^2\right] = (m_1+m_2)^2$

$$\begin{cases} m_{12}^2 = M^2+m_3^2-2ME_3 \\ m_{23}^2 = M^2+m_1^2-2ME_1 \\ m_{13}^2 = M^2+m_2^2-2ME_2 \end{cases}$$

$m_{12}^2 + m_{23}^2 = 2M^2 + m_1^2 + m_3^2 - 2M(M-E_2) =$

$= m_1^2 + m_3^2 + 2ME_2$; $m_1^2+m_3^2+2Mm_2$; $m_1^2+m_3^2+M^2+m_2^2-(m_1+m_2)^2 = M^2+m_2^2-2m_1m_3$

$\sum_i \vec{p}_i$



$\vec{p}_1+\vec{p}_2+\vec{p}_3 = 0$ $\quad |\vec{p}_1+\vec{p}_2| \le |\vec{p}_1|+|\vec{p}_2|$

disuguaglianza triangolare

$|\vec{p}_3| \le |\vec{p}_1|+|\vec{p}_2| \Rightarrow$ contorni della zona cinematicamente accessibile

$\pm|\vec{p}_1| = |\vec{p}_2| \pm |\vec{p}_3|$ $\begin{cases} + + \\ - + \\ - - \end{cases}$ tre linee nel piano (m_{12}^2, m_{23}^2)

$\sqrt{E_1^2-m_1^2}$ $\quad \sqrt{E_2^2-m_2^2}$ $\quad \sqrt{E_3^2-m_3^2}$

$M^2 = (p_K^\mu + p_p^\mu)^2$

$= m_K^2 + m_p^2 + 2E_K m_p =$

$= m_K^2 + m_p^2 + 2m_p\sqrt{p_K^2+m_K^2} = \dots =$

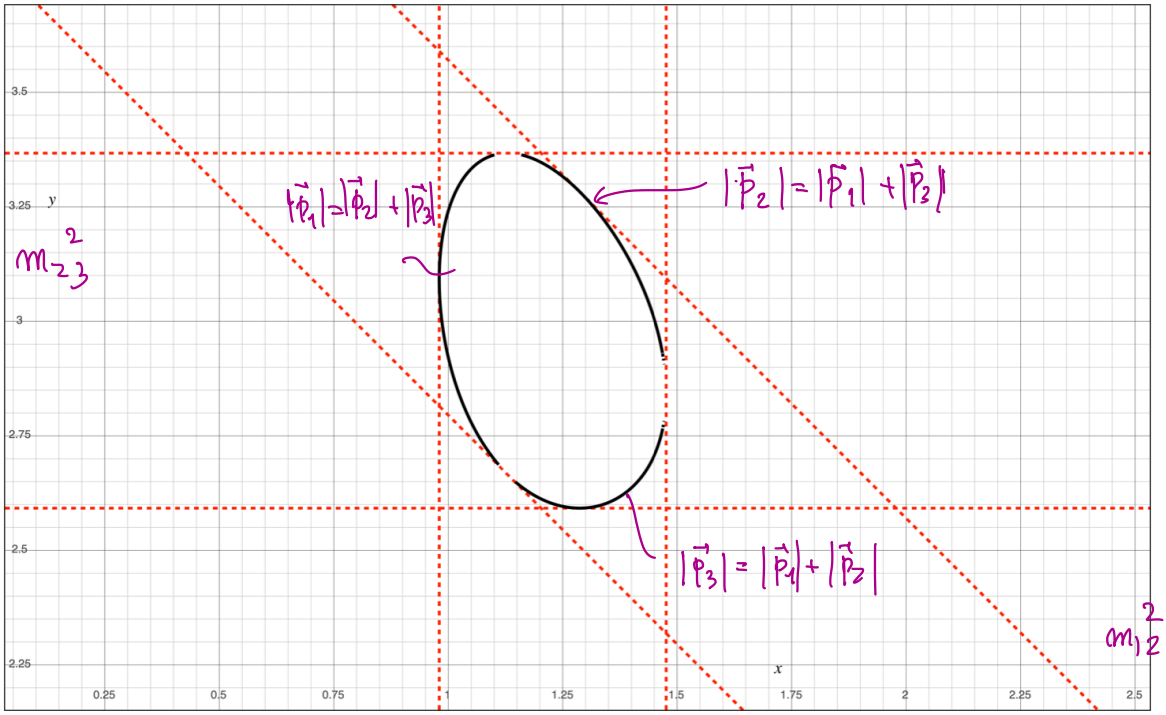
$K^- p \to K^+ K^- \Lambda$

$m_K = 0.495$ GeV $\quad p_K = 2.24$ GeV

$m_\Lambda = 1.116$ GeV

$m_p = 0.938$ GeV

$M = 2.33$ GeV



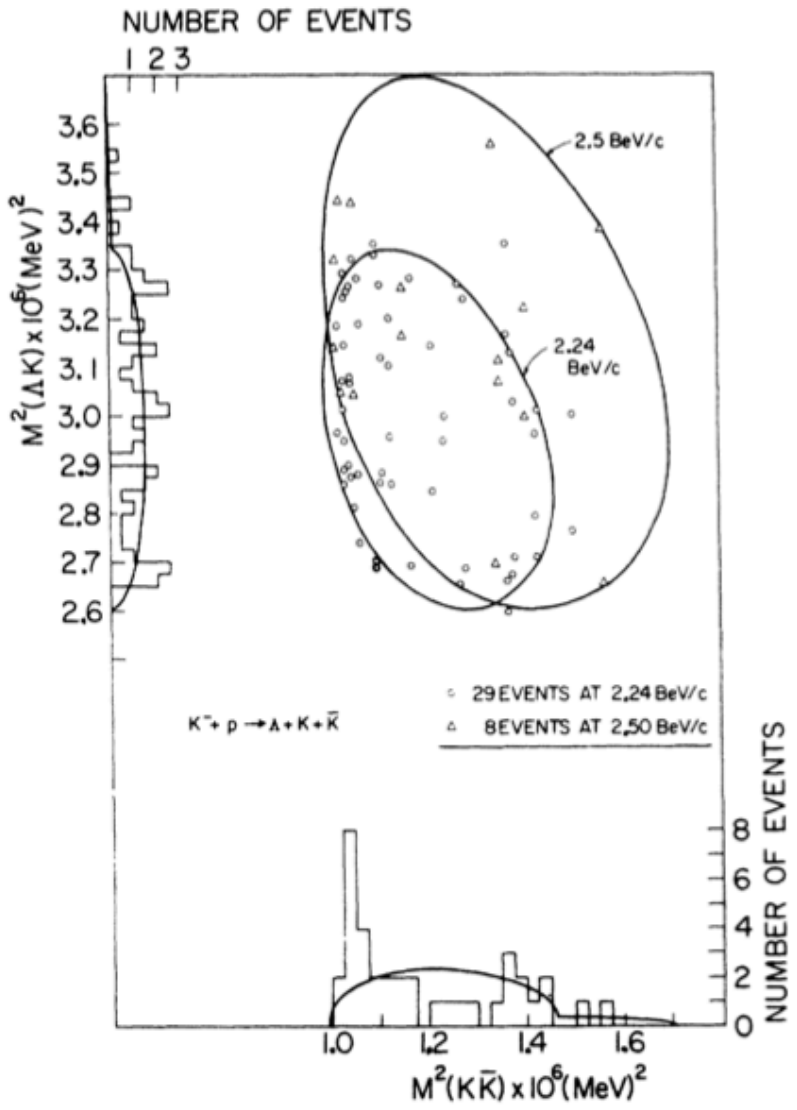


FIG. 2. The Dalitz plot for the channel $\Lambda K\bar{K}$ projected on the $M^2(K\bar{K})$ and $M^2(\Lambda K)$ axes. The solid curves on the projections are the invariant phase-space curves normalized to the total number of events.