5 Conservation/Selection rules

Reminder of conservation/selection rules
- 4-momentum
- Angular momentum
- Fermi/Bose statistics
- Charges (electric, barionic, leptonic)
- Flavor (isospin, S, C, B)
- PCT (G)

Examples
- Decay of rho0
- Decay of omega0
- Exam exercise --> with L

Conservation rules → selection of possible processes
Principle: if it is not forbidden, it will happen

- 4-momentum
- angular momentum (orbital + spin) } space-time invariance
- (A)Symmetry of total wave function → Spin-statistics
  identical { bosons → symmetric
  fermions → antisymmetric

- Charges — local gauge invariance
  electric charge + / − — U(1) QED
  color charge r g b — SU(3) QCD

- Accidental symmetries
  Baryon number B — violation not observed
  Lepton family numbers L_e L_μ L_τ approx. (neutrino oscillations)

Flavor-related
- Isospin (u-d) [STRONG]
- Strangeness, Charm, B, T [STRONG+EM]

Discrete symmetries

	C	P	T	CP	CPT	G
strong	✓	✓	✓	✓	✓	✓
EM	✓	✓	✓	✓	✓	✗
weak	✗	✗	✗	✗	✓	✗
	a lot		a little			

- **EX.** Recall $G|\pi\rangle = -|\pi\rangle$
  Particle $\rho(770)$ is a meson with $J^{PC}\, I^G = 1^{--}\, I^+$
  - How many charge states of ρ do exist?
  - Consider the (strong) decay $\rho \to \pi\pi$
  - Write the possible values of L, S, I for the two pions
  - Why is $\rho^0 \to \pi^+\pi^-$ possible while $\rho^0 \to \pi^0\pi^0$ is not?

$I = 1 \to$ 3 states $\rho^+ \rho^0 \rho^-$

$\rho \to \pi\pi \Rightarrow S = 0 \oplus 0 = 0$

$J = 1$ ($S = 0$) $L = 1$ ang-mom conservation

Strong decay → I conservation $I = 1$

Parity $P(\pi\pi) = (-1)^L = -$ ✓

Charge C. $C(\pi\pi) = (-1)^{L+S} = -$ ✓

G parity $G(\pi\pi) = (-1)^2 = +$ ✓

For $\pi^+\pi^-$, $\pi^\pm\pi^0$

$$\psi(\pi\pi) = \psi_{SPACE} \cdot \chi_{SPIN} \cdot \eta_{ISO} = \text{SYM}$$

BOSE

$L = 1$, $I = 1$ | ψ_{SPACE}: A.S., $(-1)^L$ | χ_{SPIN}: SYM | η_{ISO}: A.S., $(-1)^{I-I_1-I_2}$

Why $\rho^0 \not\to \pi^0\pi^0$?

1. Isospin $|1,0\rangle|1,0\rangle = \sqrt{\frac{2}{3}}|2,0\rangle - \sqrt{\frac{1}{3}}|0,0\rangle$

Could be EM or weak decay $\rho^0 = |1,0\rangle$

2. $C(\pi^0\pi^0) = (+)^2 = +$ ✗ could be weak decay

↳ why is it not $(-1)^{L+S}$?

IT IS !, $L \neq 1$

3. Can $L(\pi^0\pi^0) = 1$?

$\psi(\pi^0\pi^0) = \psi_{SPACE} \cdot \chi_{SPIN} = SYM$

ψ_{SPACE} SYM, χ_{SPIN} SYM

$2\pi^0$ cannot have $L=1$ -

$\rho^0 \to \pi^0\pi^0$ would violate angular momentum conservation or spin-statistics

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$\omega(782)$ $\quad I^G(J^{PC}) = 0^-(1^{--})$

only one state ω^0

$\omega \to 2\pi$ $\quad L=1, S=0$ $\quad P=-$ $\quad C=-$

$G(\pi\pi) = +$ → violates G

can be EM decay

What is the isospin state of the 2π ?

$\psi(2\pi) = \psi_{SPACE}\, \chi_{SPIN}\, \eta_{ISO} = SYM$

ψ_{SPACE} A.S. $(-1)^L$ $\quad \chi_{SPIN}$ SYM $\quad \eta_{ISO}$ ↑ MUST BE A.S.

→ $I=1$ violated

$W \to 3\pi$ $\quad S=0$, L can be 1 if

ℓ_{12} or $\ell_3 = 1$

$G(3\pi) = -1$ ✓

$P = (-1)^{\ell_{12}}(-1)^{\ell_3} = -$ $\begin{cases} \ell_{12}=1 \\ \ell_3 = 0 \end{cases}$ ~~$\begin{cases} \ell_{12}=0 \\ \ell_3 = 1 \end{cases}$~~

$C: \pi^+\pi^-\pi^0 : (-1)^{\ell_{12}} \cdot (+) = -$

$\to \ell_{12} = 1$

$C: 3\pi^0 \quad (+)^3 = + \to$ not possible (could be weak)

$W \to 2\pi \to$ EM : 1.5 % (phase space)

$W \to \pi^+\pi^-\pi^0 \to$ Strong 89 %

$W \to \pi^0\pi^0\pi^0 \to$ only weak not seen