

FUNDAMENTAL INT.

Wednesday, September 23, 2026 12:26 PM

Natural units

$$\hbar = c = 1$$

$$\hbar c = 197 \text{ MeV} \cdot \text{fm}$$

Conversion factor

$$\left[\begin{array}{l} 1 \text{ fm} = 10^{-15} \text{ m} \\ \text{"fermi"} \end{array} \right.$$

$$E \quad \text{eV} \quad \rightarrow \quad \text{eV}$$

$$m \quad \text{eV}/c^2 \quad \rightarrow \quad \text{eV}$$

$$p \quad \text{eV}/c \quad \rightarrow \quad \text{eV}$$

$$t \quad \frac{\hbar}{\text{eV}} \quad \rightarrow \quad \text{eV}^{-1}$$

$$x \quad \frac{\hbar c}{\text{eV}} \quad \rightarrow \quad \text{eV}^{-1}$$

$$\left\{ \begin{array}{l} 1 \text{ MeV}^{-1} \leftrightarrow 6.58 \cdot 10^{-22} \text{ s} \\ 1 \text{ MeV}^{-1} \leftrightarrow 197 \text{ fm} \end{array} \right.$$

Some EM quantities

$$\bullet \quad \boxed{\lambda_e = \frac{\hbar}{m_e c}} \leftrightarrow \frac{1}{m_e} \quad \text{Compton wavelength of the electron} = 386 \text{ fm}$$

It represents the localization limit of a particle:

since $\Delta x \Delta p \gtrsim \hbar$ if $\Delta x \leq \hbar/mc$ then $\Delta p \gtrsim mc$ or $\Delta E \gtrsim mc^2$ and a second particle might be created

$$\bullet \quad \boxed{r_e = \frac{1}{4\pi\epsilon_0} \frac{e^2}{m_e c^2}} = \lambda_e \alpha$$

$$\text{Classical radius of the electron} = 2.8 \cdot 10^{-15} \text{ m}$$

$$\bullet \quad \boxed{a_0 = \frac{4\pi\epsilon_0 \hbar^2}{e^2 m_e}} = \frac{\lambda_e}{\alpha}$$

$$\text{Bohr radius} = 53 \text{ pm}$$

$$\text{Where } \boxed{\alpha \equiv \frac{e^2}{4\pi\epsilon_0 \hbar c} \approx \frac{1}{137}}$$

Fine structure constant
(dimensional)

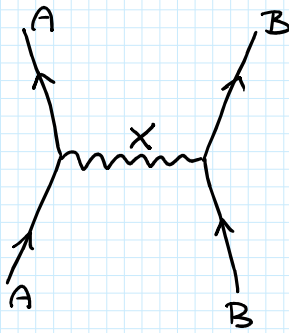
Interactions

	coupling constant	Range	Which particles	mediator
strong	$\alpha_s \approx 1$	"short"	hadrons	g (gluon)
EM	$\alpha \approx \frac{1}{137}$	"long"	charged	γ
weak	$G_F/(\hbar c)^3 \approx 1.16 \cdot 10^{-5} \text{ GeV}^{-2}$	"short"	all	$W^\pm, Z$
gravity	$G = 6.71 \cdot 10^{-39} \hbar c (\text{GeV}/c^2)^{-2}$	"long"	all	(graviton?)

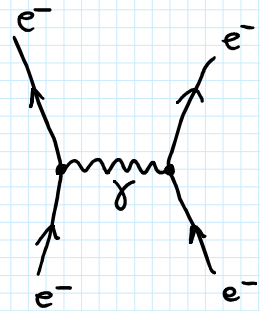
Description by exchange of a relativistic particle (relativity).

$\gamma = 6.7 \cdot 10^{-16} \text{ s} \approx 10^{-15} \text{ s}$ (very long) $\approx 10^{-15} \text{ s}$

Description by exchange of a mediator particle (relativity):



i.e. electromagnetism



Interaction range is related to the mass of the mediator particle
In CM of A:

$$A(m_A, \vec{0}) \rightarrow A(E_A', \vec{p}') + X(E_X', -\vec{p}') \Rightarrow E_X' = m_A - E_A' < 0 ?$$

since $E_X' \geq m_X c^2$ one may think that energy conservation is "violated" for a short time such that $\Delta E \Delta t \sim \hbar$
if $\Delta E \sim m_X c^2$ $\Delta t \sim \hbar / m_X c^2$ during which X travels at most $\Delta x \sim \hbar c / m_X c^2 = \lambda_X$

$$\text{Range} \sim \frac{\hbar c}{m_X}$$

(but energy conservation is not violated)
X is a "virtual particle"

- Strong interactions: $R \sim 1 \text{ fm}$ (proton radius) $\rightarrow \Rightarrow m_\pi \sim 197 \text{ MeV}$ (Yukawa)
- EM: $m_\gamma = 0 \Rightarrow R = \infty$

Better explanation:

QM: $\psi(\vec{r}, x) = N e^{i(\vec{p} \cdot \vec{r} - Et) / \hbar}$ $\begin{cases} E = \hbar \nu \\ \lambda = \hbar / p \end{cases}$

with $E = \frac{p^2}{2m}$

$$\Rightarrow i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi \quad \text{Schrödinger}$$

In relativity $E^2 = \vec{p}^2 c^2 + m^2 c^4$

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = -\hbar^2 c^2 \nabla^2 \psi + m^2 c^4 \psi$$

Klein-Gordon

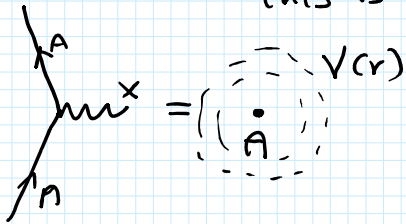
Also written $(\square + m^2)\psi = 0$

where $\square \equiv \frac{\partial^2}{\partial t^2} - \nabla^2$ (natural units)

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Connection to classical physics: if A is very heavy and does not move this is equivalent to a static potential



$$\text{Klein-Gordon} \Rightarrow \nabla^2 \psi(\vec{r}) = \frac{m_x^2 c^2}{\hbar^2} \psi(\vec{r})$$

where $\psi(\vec{r}) = V(\vec{r})$ is the potential

- e.g. if $m_x = 0$ $\nabla^2 V(r) = 0 \Rightarrow V(\vec{r}) = \left(\frac{-e^2}{4\pi\epsilon_0} \right) \frac{1}{r}$ (Coulomb)

- if $m_x \neq 0$ solution is $V(\vec{r}) = \left(\frac{-g^2}{4\pi} \right) \frac{e^{-r/R}}{r}$

Yukawa potential

where $R \equiv \hbar/m_x c$

Note: the constants are so far arbitrary

the potential decreases with r much faster than Coulomb
"short range,"