

- Decay
- Collisions
- Bound states

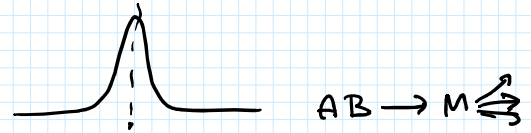
kinematics

- $\begin{cases} AB \rightarrow AB & \text{elastic} \\ AB \rightarrow X & \text{inelastic} \end{cases}$

- mass $\begin{cases} \text{formation} \\ \text{production} \end{cases}$

$\sigma(E)$

$AB \rightarrow MX$



$$m^2 = (\sum_i p_i)^2$$

invariant mass

- $AB \rightarrow CD \quad (2 \rightarrow 2)$

of quantities $16 - 4 \text{ masses} - 4 \text{ conservation} = 8 \text{ quantities}$

p_A, p_B, p_C, p_D

$$E_A^2 = \vec{p}_A^2 + m_A^2$$

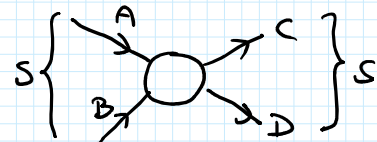
$$\vec{p}_A + \vec{p}_B = \vec{p}_C + \vec{p}_D$$

Invariants: $3 p_i \rightarrow p_i^2 \quad p_i \cdot p_j = 4 \text{ masses} + 2$

$i=1,3 \quad (3) \quad \frac{3-2}{2} = 3$

$$\begin{cases} s \equiv (p_A + p_B)^2 \\ t \equiv (p_A - p_C)^2 \\ u \equiv (p_A - p_D)^2 \end{cases}$$

Mandelstam Variables



$$(\vec{E}_A^x + \vec{E}_B^x, \vec{0})^2 = (\vec{E}_A^x + \vec{E}_B^x)^2 \quad (\text{CM})$$

$$(\vec{E}_A^L + m_B, \vec{p}_A^L + \vec{0})^2 \quad (\text{LAB})$$

$$\begin{aligned} p_A^2 + p_B^2 + 2 p_A p_B &= m_A^2 + m_B^2 + 2 (\vec{E}_A^L, \vec{p}_A^L) (m_B, \vec{0}) \\ &= m_A^2 + m_B^2 + 2 E_A^L m_B \end{aligned}$$

if $m_A = m_B$: $s = 2 m_A (E_A^L + m_A)$

• collider

$$\vec{E}_A^x \leftarrow \vec{E}_A^x$$

CM $\equiv$ LAB

$$M = 2 E_A^x$$

• fixed target

$$\begin{cases} M^2 = 2 m_A (E_A^L + m_A) \\ M = \sqrt{2 m_A (E_A^L + m_A)} \end{cases}$$

- ② t for elastic process:

$$AB \rightarrow A'B'$$

$$\vec{p}_A + \vec{p}_B = \vec{p}_A' + \vec{p}_B'$$

$$t = (\vec{p}_A - \vec{p}_A')^2 \quad (\text{four-momentum transfer})^2 = -t$$

$$= (\vec{p}_B' - \vec{p}_B)^2$$

③ u (for elastic process):

$$u = (\vec{p}_A - \vec{p}_B')^2$$

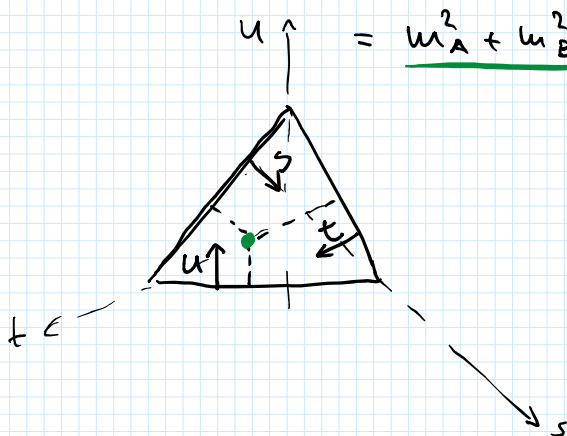
$$= (\vec{p}_B - \vec{p}_A')^2$$

$$\underline{s + t + u} = (\vec{p}_A + \vec{p}_B)^2 + (\vec{p}_A - \vec{p}_C)^2 + (\vec{p}_A - \vec{p}_D)^2 =$$

$$m_A^2 + m_B^2 + 2\vec{p}_A \cdot \vec{p}_B + m_A^2 + m_C^2 - 2\vec{p}_A \cdot \vec{p}_C + m_A^2 + m_D^2 - 2\vec{p}_A \cdot \vec{p}_D$$

$$3m_A^2 + m_B^2 + m_C^2 + m_D^2 + 2\vec{p}_A \cdot (\underbrace{\vec{p}_B - \vec{p}_C - \vec{p}_D}_{-\vec{p}_A})$$

$$u = \underline{m_A^2 + m_B^2 + m_C^2 + m_D^2}$$



• $s \geq 0$ always

• $t \leq 0$ $u \leq 0$ usually

ex. elastic $\begin{cases} A=C \\ B=D \end{cases}$

"Breit" "brick-wall" frame $\vec{p}_A = -\vec{p}_C = -\vec{p}_A'$ ($\vec{p}_A + \vec{p}_C = 0$)

$$t = (\vec{p}_A - \vec{p}_C)^2 = (\cancel{E_A - E_C}, \vec{p}_A - \vec{p}_C)^2 = 0 - 4\vec{p}_A^2 \leq 0$$

$E_A = E_C \leftarrow$

• Elastic: $|\vec{p}_A^*| = |\vec{p}_B^*| = |\vec{p}_C^*| = |\vec{p}_D^*|$

$\underbrace{\quad}_{|\vec{p}_i^*|} \quad \underbrace{\quad}_{|\vec{p}_f^*|}$

• $S = (E_A^* + E_B^*)^2 = E_A^{*2} + E_B^{*2} + 2E_A^* E_B^* \quad (\text{CM system})$

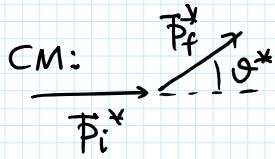
$$= m_A^2 + \vec{p}_i^{*2} + m_B^2 + \vec{p}_i^{*2} + 2E_A^* E_B^*$$

$$\geq m_A^2 + m_B^2 + 2m_A m_B = (m_A + m_B)^2$$

$$\underline{\sqrt{S} \geq m_A + m_B}$$

$$\sqrt{s} \geq m_A + m_B$$

$$\begin{aligned}
 \bullet \quad t &= (E_A - E_C, \vec{p}_A - \vec{p}_C)^2 = -(\vec{p}_A - \vec{p}_C)^2 \\
 &= -\vec{p}_A^2 - \vec{p}_C^2 + 2\vec{p}_A \cdot \vec{p}_C \\
 &= -2\vec{p}_i^2 + 2\vec{p}_i^2 \cos \theta^* \\
 &= -2\vec{p}_i^2 (1 - \cos \theta^*) \\
 &= -4\vec{p}_i^2 \sin^2(\theta^*/2) \quad (\text{CM system})
 \end{aligned}$$



{ Decay: $1 \rightarrow N$
 { Collision: $2 \rightarrow N$

Decay: In CM: $N(t) = N(0) e^{-t/\tau}$ $\frac{dN}{dt} = -N/\tau$

Decay width $\Gamma = \frac{\hbar}{\tau}$

More decay modes Γ_i
($i = 1 \dots k$)

Partial decay widths

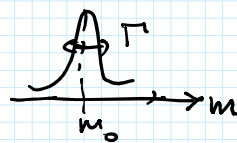
$$\Gamma = \sum_{i=1}^k \Gamma_i$$

Branching ratio $BR_i = \frac{\Gamma_i}{\Gamma}$

$$\sum BR_i = 1$$

$$\Gamma_i = \Gamma \cdot BR_i = \frac{BR_i \hbar}{\tau}$$

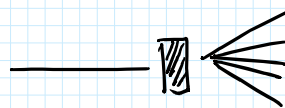
Unstable $\rightarrow \Delta E \rightarrow \Delta m$
($\tau \neq 0$)



$\tau \sim 10^{-6} \div 10^{-12} \text{ s}$ (weak)
 $10^{-15} \div 10^{-20} \text{ s}$ (EM)
 $\leq 10^{-20} \text{ s}$ (strong)

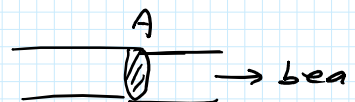
ℓ
 path length
 $\ell = \gamma \beta c \tau$

Collisions: cross section



$$\text{Intensity} = \frac{n \cdot \text{particles}}{t}$$

$$\Phi = \frac{n \cdot \text{particles}}{t \cdot A}$$



cross section N : scattering centres, $n = \frac{N}{V}$ (density)

Interaction rate $R = \sigma \Phi N$ if Φ constant ("thin target")

$$[R] = \frac{1}{T} = \frac{[\sigma][\Phi][N]}{L^2 \cdot \frac{1}{TL^2} \cdot 1}$$

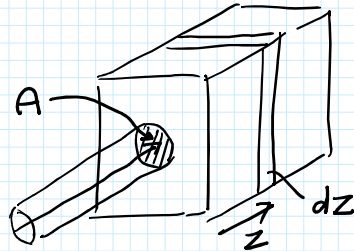
$$1 \text{ barn} = 10^{-28} \text{ m}^2$$

$$(1 \text{ mb} = 10^{-3} \text{ b})$$

• More reactions:

σ_i ($i=1 \dots k$) $\sum_i \sigma_i = \sigma \leftarrow$ total cross section

• If target is thick $\rightarrow \Phi$ changes along z :



$I(z) = \Phi \cdot A$ In a dz thickness:

$$dI = -dR = -\sigma \Phi dN$$

$$= -\sigma \frac{I}{A} n A dz$$

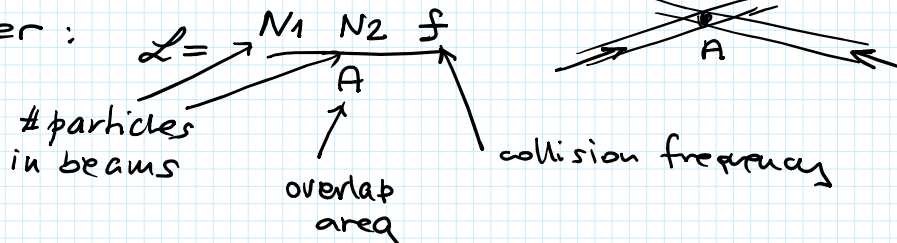
$$I(z) = I_0 e^{-z/\lambda}$$

$$\lambda = \frac{1}{\sigma n}$$

Interaction length

• Luminosity $\mathcal{L} = \frac{R}{\sigma} = \Phi \cdot N = \frac{I N}{A}$ $[\mathcal{L}] = \frac{1}{T L^2}$

In a collider:



• Integrated luminosity

$$\int \mathcal{L} dt \quad [\mathcal{L} dt] = \frac{1}{L^2} \text{ (barn}^{-1} \text{ b}^{-1})$$

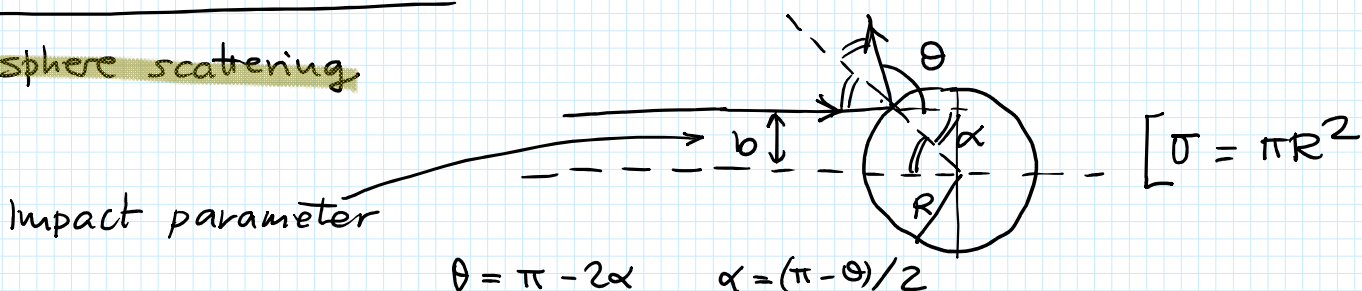
$$\mathcal{L} dt = 100 \text{ pb}^{-1} \Rightarrow 100 \text{ events for a } \sigma = 1 \text{ pb}$$

• Differential cross-section

$$\frac{d\sigma}{dx}$$

$$d\Omega, dE, \dots, d\vec{p}_1, d\vec{p}_2, \dots$$

Hard sphere scattering



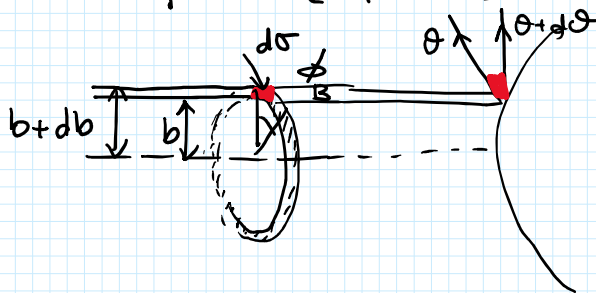
$$\theta = \pi - 2\alpha \quad \alpha = (\pi - \theta)/2$$

Impact parameter

$$\theta = \pi - 2\alpha \quad \alpha = (\pi - \theta)/2$$

$$b = R \sin \alpha = R \sin(\pi/2 - \theta/2) = R \cos(\theta/2)$$

$$\text{if } b \in [b, b+db] \rightarrow \theta \in [\theta, \theta+d\theta]$$



$$\begin{cases} d\sigma = |b d\phi \cdot db| \\ d\Omega = |d\phi \sin\theta d\theta| \end{cases}$$

$$\frac{d\sigma}{d\Omega} = \left| \frac{b}{\sin\theta} \frac{db}{d\theta} \right|$$

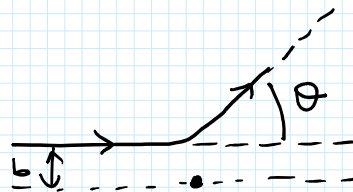
$$\text{if } b(\theta) \text{ known} \Rightarrow d\sigma/d\Omega$$

For hard sphere:

$$\frac{db}{d\theta} = -\frac{R}{2} \sin(\theta/2) \Rightarrow \frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta} \frac{R}{2} \sin(\theta/2) = \frac{R^2 \sin\theta/2 \cos\theta/2}{2(2 \sin\theta/2 \cos\theta/2)} = \frac{R^2}{4}$$

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \frac{R^2}{4} 4\pi = \pi R^2$$

Rutherford scattering



$$b = \frac{q_1 q_2}{2E_K} \cot(\theta/2)$$

↑
kinetic energy

$$\frac{db}{d\theta} = \frac{q_1 q_2}{2E_K} \left(-\frac{1}{\sin^2(\theta/2)} \frac{1}{2} \right)$$

$$\frac{d\sigma}{d\Omega} = \frac{q_1 q_2}{2E_K} \frac{\cot(\theta/2)}{\sin\theta} \frac{1}{\sin\theta} \frac{q_1 q_2}{2E_K} \left(\frac{1}{\sin^2(\theta/2)} \frac{1}{2} \right) = \left[\frac{q_1 q_2}{4E_K} \frac{1}{\sin^2(\theta/2)} \right]^2$$

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \infty \quad \text{infinite range of EM interaction}$$

$$V(r) \propto \frac{1}{r} \quad \text{from } \frac{d\sigma}{d\Omega} \text{ measurement}$$