

• Ex. Parity of deuteron (deuteron nucleus)

$d = pn$ (spin 1)

$s = 1/2 \rightarrow s = 1/2 \rightarrow S = 0, 1$

lowest-energy states usually are $L=0$

$J=1, L=0 \Rightarrow S=1 \quad 3S_1 \quad P(d) = +1$

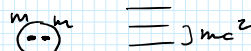
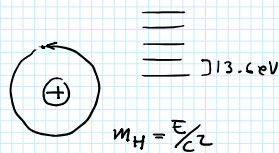
Check: magnetic moments should add if spins are parallel:

$\mu_d = (2.793 - 1.913) \mu_N = 0.88 \mu_N$

experiment: $\mu_d = 0.857 \mu_N$ almost ok.

Not exactly because L not really a good quantum number
d also contains a fraction of $3D_1$

$\frac{e\hbar}{2m_p}$
(Nuclear magneton)



Bound states for which energy level differences are of same order as mass (ground state) are called different particles

$\gamma \rightarrow e^+e^-$

$\pi^0 \rightarrow e^+e^-e^-$

• Ex. Parity of π^0 (spin 0)

$\pi^0 \rightarrow \gamma\gamma$ EM process, P conserved $P(\pi^0) = P(\gamma\gamma)$

Available vectors: $\vec{k}, \vec{E}_1, \vec{E}_2$ (with $\vec{k} \cdot \vec{E}_i = 0$, transverse pol.)
 γ momentum γ polarization vectors

M must be linear and symmetric in $\vec{E}_1, \vec{E}_2$ (scalar, $P = +1$)

the variables of the two γ , and have $J=0$ $\vec{k}(\vec{E}_1 \times \vec{E}_2)$ (pseudoscalar, $P = -1$)

If $\gamma \rightarrow e^+e^-$ the decay plane of e^+e^- tends to be $\parallel$ to $\vec{E}$

Studying $\pi^0 \rightarrow \gamma\gamma \rightarrow e^+e^-e^+e^-$ the two planes are $\perp \rightarrow \propto \vec{E}_1 \times \vec{E}_2$

$\Rightarrow P(\pi^0) = -1$

• Ex. Parity of π^- (spin 0)

$\pi^- d \rightarrow nn$ with $L(\pi^- d) = 0$ (known)

Recall that $J^P(d) = 1^+$ $J(\pi^- d) = 1$

strong int. $\rightarrow P$ conserved $\rightarrow P(\pi^- d) = P(nn)$

$P(\pi^- d) = P(\pi^-)(+1)(-1)^L = P(\pi^-)$

$P(nn) = (-1)^L$ but $(*)$ exchange = $(-1)^L(-1)^{S+1}$ must be (-1)

$\rightarrow L+S = \text{even}$

(*) see later

$S=0, L \text{ even} \rightarrow \text{cannot have } J=1$

$S=1, L \text{ odd} \rightarrow L=1 \text{ to have } J=1$

so $P(nn) = (-1)^L = -1$

$P(\pi^-) = -1$

C : charge conjugation

Inverts the sign of all charges (including baryon number, lepton number, etc.)

$\begin{cases} \Phi \xrightarrow{C} -\Phi \\ B \xrightarrow{C} -B \\ L \xrightarrow{C} -L \end{cases}$

$C|\vec{p}, \vec{s}, \{\Phi\}\rangle = |\vec{p}, \vec{s}, \{-\Phi\}\rangle$
all charges

$C^2 = 1 \quad C|\Phi\rangle = \lambda|\Phi\rangle \quad \lambda^2 = 1 \quad \lambda = \pm 1$ eigenvalues

But only totally neutral states can be eigenstates of C

$C|e^+\rangle = |e^+\rangle \neq |e^-\rangle$

$C|\gamma\rangle = \lambda|\gamma\rangle$

$C|n\rangle = |\bar{n}\rangle \neq |n\rangle$

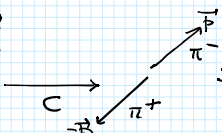
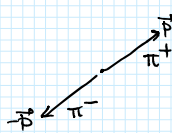
(Baryon number)

$C|\gamma\rangle = -|\gamma\rangle$ so that

$H \propto \Phi A^\mu$ is symmetric under C , as experimentally EM respects C symmetry

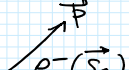
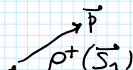
• Composite states:

no spin
 $J(\pi^\pm) = 0$



same as P , so $C = (-1)^L$

spin $\neq 0$
 $J^P = 1^-$



C is not

$C|n\gamma\rangle = (-1)^n |n\gamma\rangle$

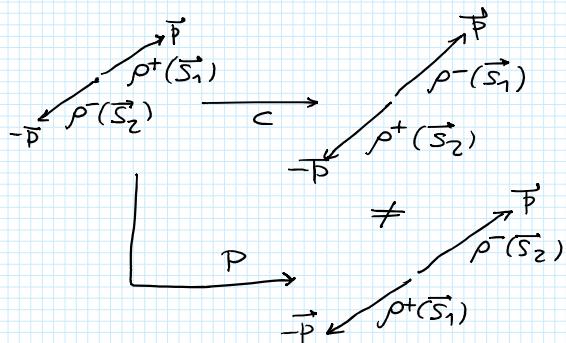
$\pi^0 \rightarrow \gamma\gamma$

$C = + \quad C = +$

$C|\pi^0\rangle = +|\pi^0\rangle$

$J^{PC}(\pi^0) = 0^{-+}$

spin $\neq 0$
 $J(p^\pm) = 1$



C is not the same as P, one should also exchange spins

$C = +$ $C = +$
 $C|\pi^0\rangle = +|\pi^0\rangle$
 $C|\pi^\pm\rangle = |\pi^\mp\rangle$

$J^{PC}(\pi^0) = 0^{-+}$

$\pi^0 \rightarrow \gamma\gamma$
 $\frac{\Gamma(\pi^0 \rightarrow \gamma\gamma)}{\Gamma(\pi^0 \rightarrow 2\gamma)} < 10^{-5}$

Recall:

Combination of (J_1, m_1) and (J_2, m_2) to form (J, M) :

$|J_1, J_2; J, M\rangle = \sum_{m_1+m_2=M} \underbrace{\langle J_1, m_1; J_2, m_2 | J, M \rangle}_{\text{Clebsch-Gordan coefficients}} |J_1, m_1\rangle |J_2, m_2\rangle$
 $M = m_1 + m_2$

For $J_1 = J_2 = 1$:

$|1, 1; 0, 0\rangle = \frac{1}{\sqrt{3}} |1, 1\rangle |1, -1\rangle - \frac{1}{\sqrt{3}} |1, 0\rangle |1, 0\rangle + \frac{1}{\sqrt{3}} |1, -1\rangle |1, 1\rangle$ symmetric

$\begin{cases} |1, 1; 1, -1\rangle = -\frac{1}{\sqrt{2}} |1, 1\rangle |1, 0\rangle + \frac{1}{\sqrt{2}} |1, 0\rangle |1, -1\rangle \\ |1, 1; 1, 0\rangle = \frac{1}{\sqrt{2}} |1, 1\rangle |1, -1\rangle - \frac{1}{\sqrt{2}} |1, -1\rangle |1, 1\rangle \\ |1, 1; 1, 1\rangle = \dots \end{cases}$ antisymmetric

$\begin{cases} |1, 1; 2, -2\rangle = |1, -1\rangle |1, -1\rangle \\ \dots \\ \dots \end{cases}$ symmetric

For spin J : $(-1)^{J-J_1-J_2}$ under exchange

so for $J_1 = J_2 = \text{integer}$, spin exchange $= (-1)^J$

$C = (-1)^{L+S}$

Fermion - antifermion:

Ex. $\frac{1}{2} \otimes \frac{1}{2}$

$|\frac{1}{2}, \frac{1}{2}; 0, 0\rangle = \frac{1}{\sqrt{2}} |\frac{1}{2}, \frac{1}{2}\rangle |\frac{1}{2}, -\frac{1}{2}\rangle - \frac{1}{\sqrt{2}} |\frac{1}{2}, -\frac{1}{2}\rangle |\frac{1}{2}, \frac{1}{2}\rangle$ antisymmetric

$\begin{cases} |\frac{1}{2}, \frac{1}{2}; 1, -1\rangle = |\frac{1}{2}, -\frac{1}{2}\rangle |\frac{1}{2}, -\frac{1}{2}\rangle \\ |\frac{1}{2}, \frac{1}{2}; 1, 0\rangle = \frac{1}{\sqrt{2}} |\frac{1}{2}, \frac{1}{2}\rangle |\frac{1}{2}, -\frac{1}{2}\rangle + \frac{1}{\sqrt{2}} |\frac{1}{2}, -\frac{1}{2}\rangle |\frac{1}{2}, \frac{1}{2}\rangle \\ |\frac{1}{2}, \frac{1}{2}; 1, 1\rangle = \dots \end{cases}$ symmetric

$J_1 = J_2 = \text{half-integer}$: exchange $(-1)^{J+1}$

but $P = (-1)^{L+1}$ in this case so again

$C = (-1)^{L+S}$

T time reversal

$t \rightarrow -t$ and

$\vec{P} \rightarrow -\vec{P}$
 $\vec{L} = \vec{r} \times \vec{p} \rightarrow -\vec{L}$
 $\vec{S} \rightarrow -\vec{S}$

is antilinear (transforms numbers in their complex conjugates)

Has no eigenvalues.

If T symmetry is valid:

$A(\vec{P}_A, m_A) + B(\vec{P}_B, m_B) \rightarrow C(\vec{P}_C, m_C) + D(\vec{P}_D, m_D)$

is equal to

$C(-\vec{P}_C, -m_C) + D(-\vec{P}_D, -m_D) \rightarrow A(-\vec{P}_A, -m_A) + B(-\vec{P}_B, -m_B)$

CPT theorem : in quantum field theory, under very general assumptions (Lorentz, spin-statistics, ...)

The product of C, P, T (in any order) is a symmetry

Indeed experimentally:

	C	P	T	CP	CPT
strong	✓	✓	✓	✓	✓
EM	✓	✓	✓	✓	✓
weak	✗	✗	✗	✗	✓

a lot
a little

Consequences of CPT :
(and also of C, but this is violated)

$$\begin{cases} m(a) = m(\bar{a}) \\ \tau(a) = \tau(\bar{a}) \\ \vec{\mu}(a) = -\vec{\mu}(\bar{a}) \end{cases}$$

A few things to remember

- A massless particle can only have $S_z = \pm 1$ along its direction of motion: photon has spin 1 but not $S_z = 0$

- Landau-Yang theorem: a spin 1 particle cannot decay in 2γ

spin 1 $\nrightarrow 2\gamma$ Why?

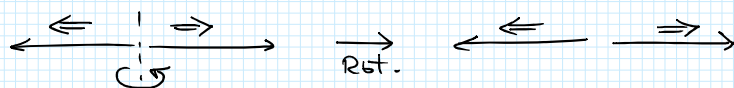
Photon: $\Rightarrow$ or $\Leftarrow$ so for 2γ : $\Leftarrow \Rightarrow$ $\Rightarrow \Leftarrow$

therefore $J_z = 0$ or 2 along direction of motion.

If $J=1$: J_z must be 0, so a state $\propto Y_1^0(\theta, \varphi) \propto \cos\theta$

which for a rotation by π around an axis $\perp z$ ($\theta \rightarrow \pi - \theta$) gives a (-1)

But such rotation = exchange of photons



and the sign must be $(+1)$, identical bosons

$\pi^0 \rightarrow 2\gamma \Rightarrow$ spin of π^0 cannot be 1

$\rho^0 \not\rightarrow 2\gamma$ because spin of ρ^0 is 1

$H \rightarrow 2\gamma$ Higgs cannot have spin 1

Principle of detailed balance

If T is valid:

$$\mathcal{M}[A(\vec{p}_A, m_A) + B(\vec{p}_B, m_B) \rightarrow C(\vec{p}_C, m_C) + D(\vec{p}_D, m_D)]$$

$$= \mathcal{M}[C(-\vec{p}_C, -m_C) + D(-\vec{p}_D, -m_D) \rightarrow A(-\vec{p}_A, -m_A) + B(-\vec{p}_B, -m_B)]$$

If also P is valid:

$$= \mathcal{M}[C(\vec{p}_C, -m_C) + D(\vec{p}_D, -m_D) \rightarrow A(\vec{p}_A, -m_A) + B(\vec{p}_B, -m_B)]$$

If we ignore spin (i.e. average over all spin orientations, i.e. not measure it)

Spin average of $A(\vec{p}_A) + B(\vec{p}_B) \rightarrow C(\vec{p}_C) + D(\vec{p}_D)$

is equal to spin average of $C(\vec{p}_C) + D(\vec{p}_D) \rightarrow A(\vec{p}_A) + B(\vec{p}_B)$

same reaction in the two directions

Transformation of different quantities under C, P, T:

	C	P	T
$\vec{r}$	$\vec{r}$	$-\vec{r}$	$\vec{r}$
t	t	t	$-t$
$\vec{L}$	$\vec{L}$	$-\vec{L}$	$-\vec{L}$
$\vec{S}$	$\vec{S}$	$-\vec{S}$	$-\vec{S}$
$\vec{p}$	$\vec{p}$	$-\vec{p}$	$-\vec{p}$
ϕ	$-\phi$	ϕ	ϕ
B	$-B$	B	B