

APPLICATIONS OF ISOSPIN CONSERVATION

- Ex.** deuteron d (= proton + neutron): only bound state of nucleons $N = (p, n)$

$$|NN\rangle : 2 \otimes 2 = 1 \oplus 3$$

isodoublet

$$I(N) = 1/2$$

$d = pn : I_3 = 0$ $I?$ must be 0 (otherwise also pp, nn would be bound states)

$$|d\rangle = |0,0\rangle = \frac{1}{\sqrt{2}} (|p\rangle|n\rangle - |n\rangle|p\rangle)$$

- $\pi^0 \quad \pi^\pm$
135 MeV 139 MeV

$$\begin{matrix} |1,-1\rangle & |1,0\rangle & |1,+1\rangle \\ \pi^- & \pi^0 & \pi^+ \end{matrix}$$

$$I(\pi) = 1$$

pions are an isotriplet

- Ex.** Consider the reactions:

$$\begin{matrix} a & \left\{ \begin{array}{l} pp \rightarrow d\pi^- \\ pn \rightarrow d\pi^0 \\ nn \rightarrow d\pi^+ \end{array} \right. \end{matrix} \quad (I=0) \otimes (I=1) \Rightarrow (I=1)$$

H (strong) conserves isospin: I must be the same in initial and final states (and of course also I_3)

$$(a): \frac{\langle 0,0 | \langle 1,1 |}{\langle 1,1 |} H \frac{|1/2, 1/2\rangle |1/2, 1/2\rangle}{|1,1\rangle} = \mathcal{M}_a$$

$$(b): \frac{\langle 0,0 | \langle 1,0 |}{\langle 1,0 |} H \frac{|1/2, 1/2\rangle |1/2, -1/2\rangle}{\frac{1}{\sqrt{2}} (|1,0\rangle - |0,0\rangle)} = \frac{\langle 1,0 | H | 1,0 \rangle}{\sqrt{2}} = \mathcal{M}_b$$

$$(c): \frac{\langle 0,0 | \langle 1,-1 |}{\langle 1,-1 |} H \frac{|1/2, -1/2\rangle |1/2, -1/2\rangle}{|1,-1\rangle} = \mathcal{M}_c$$

$$[H, I_3] = 0 \Rightarrow [H, I_\pm] = 0 \quad \text{no dependence on } I_3$$

$$\langle I, \alpha | H | I, \alpha \rangle = \langle I, \beta | H | I, \beta \rangle$$

$$\mathcal{M}_a = \mathcal{M}_c = \mathcal{M}_b \sqrt{2}$$

$$\begin{cases} \sigma(pp \rightarrow d\pi^+) = \sigma(nn \rightarrow d\pi^-) \\ \sigma(pp \rightarrow d\pi^+) = 2\sigma(pn \rightarrow d\pi^0) \end{cases}$$

- Ex.** πN
Consider the reactions:

$$\begin{matrix} \pi^+ p \rightarrow \pi^+ p & \pi^+ n \rightarrow \pi^+ n \\ \pi^0 p \rightarrow \pi^0 p & \pi^0 n \rightarrow \pi^0 n \\ \pi^- p \rightarrow \pi^- p & \pi^- n \rightarrow \pi^- n \end{matrix}$$

elastic

$$\left. \begin{matrix} \pi^+ n \rightarrow \pi^0 p \\ \pi^0 p \rightarrow \pi^+ n \\ \pi^0 n \rightarrow \pi^- p \\ \pi^- p \rightarrow \pi^0 n \end{matrix} \right\} \text{ "charge exchange" }$$

$$\begin{matrix} 1/2 & 1 & 1/2 \\ \pi & N & \end{matrix}$$

$$I=1 \otimes I=1/2 = \begin{cases} I=1/2 \\ I=3/2 \end{cases}$$

$$\begin{array}{lcl}
 | \pi^+ p \rangle & \pi & N \\
 | \pi^0 p \rangle & 0 & +1/2 : \\
 | \pi^- p \rangle & -1 & +1/2 : \\
 | \pi^+ n \rangle & +1 & -1/2 : \\
 | \pi^0 n \rangle & 0 & -1/2 : \\
 | \pi^- n \rangle & 1 & -1/2 :
 \end{array}
 \begin{array}{l}
 +1/2 : \\
 +1/2 : \\
 +1/2 : \\
 -1/2 : \\
 -1/2 : \\
 -1/2 :
 \end{array}
 \begin{array}{l}
 | 3/2, +3/2 \rangle \\
 \sqrt{2/3} | 3/2, 1/2 \rangle - \frac{1}{\sqrt{3}} | 1/2, 1/2 \rangle \\
 \frac{1}{\sqrt{3}} | 3/2, -1/2 \rangle - \sqrt{2/3} | 1/2, -1/2 \rangle \\
 \frac{1}{\sqrt{3}} | 3/2, 1/2 \rangle + \sqrt{2/3} | 1/2, 1/2 \rangle \\
 \sqrt{2/3} | 3/2, -1/2 \rangle + \frac{1}{\sqrt{3}} | 1/2, -1/2 \rangle \\
 | 3/2, -3/2 \rangle
 \end{array}$$

$$I=1 \otimes I=1/2 = \begin{cases} I=1/2 \\ I=3/2 \end{cases}$$

$$\text{elastic} \left\{ \begin{array}{l}
 \mathcal{M}(\pi^+ p \rightarrow \pi^+ p) = \mathcal{M}_{3/2} \\
 \mathcal{M}(\pi^0 p \rightarrow \pi^0 p) = \frac{2}{3} \mathcal{M}_{3/2} + \frac{1}{3} \mathcal{M}_{1/2} \\
 \mathcal{M}(\pi^- p \rightarrow \pi^- p) = \frac{1}{3} \mathcal{M}_{3/2} + \frac{2}{3} \mathcal{M}_{1/2} \\
 \mathcal{M}(\pi^+ n \rightarrow \pi^+ n) = \frac{1}{3} \mathcal{M}_{3/2} + \frac{2}{3} \mathcal{M}_{1/2} \\
 \mathcal{M}(\pi^0 n \rightarrow \pi^0 n) = \frac{2}{3} \mathcal{M}_{3/2} + \frac{1}{3} \mathcal{M}_{1/2} \\
 \mathcal{M}(\pi^- n \rightarrow \pi^- n) = \mathcal{M}_{3/2}
 \end{array} \right.$$

$$\bullet \pi^+ n \rightarrow \pi^0 p \quad |1,1\rangle |1/2, -1/2\rangle \rightarrow |0,0\rangle |1/2, 1/2\rangle$$

$$\left(\sqrt{2/3} \langle 3/2, 1/2 | - \frac{1}{\sqrt{3}} \langle 1/2, 1/2 | \right) \mathcal{H} \left(\frac{1}{\sqrt{3}} | 3/2, 1/2 \rangle + \sqrt{2/3} | 1/2, 1/2 \rangle \right)$$

$$\mathcal{M}(\pi^+ n \rightarrow \pi^0 p) = \sqrt{2/3} \mathcal{M}_{3/2} - \sqrt{2/3} \mathcal{M}_{1/2}$$

$$\bullet \pi^0 n \rightarrow \pi^- p$$

$$\left(\frac{1}{\sqrt{3}} \langle 3/2, -1/2 | - \sqrt{2/3} \langle 1/2, -1/2 | \right) \mathcal{H} \left(\sqrt{2/3} | 3/2, -1/2 \rangle + \frac{1}{\sqrt{3}} | 1/2, -1/2 \rangle \right)$$

$$\mathcal{M}(\pi^0 n \rightarrow \pi^- p) = \sqrt{2/3} \mathcal{M}_{3/2} - \sqrt{2/3} \mathcal{M}_{1/2} = \mathcal{M}(\pi^- p \rightarrow \pi^0 n)$$

• same for other two

• In some cases one of the two amplitudes dominates, e.g. close to a resonance

$$\pi p \rightarrow \Delta \rightarrow \pi p \quad \Delta(1232) \quad I=3/2$$

$$\mathcal{M}_{3/2} \gg \mathcal{M}_{1/2} \quad \text{when } E_{\text{cm}} \simeq m_{\Delta} = 1232 \text{ MeV}$$

$$\frac{\sigma_{\pi^+ p}}{\Gamma_{\pi^+ p}} = \frac{2}{9}$$

$$\sigma(\pi^+ p \rightarrow \pi^+ p) \approx 9 \sigma(\pi^- p \rightarrow \pi^- p)$$

$$\sigma(\pi^- p \rightarrow \pi^0 n) \approx \frac{2}{9} \sigma(\pi^+ p \rightarrow \pi^+ p)$$

$$E_{\text{cm}} \simeq 1232 \text{ MeV}$$

$$\sigma(\pi^- p \rightarrow \pi^0 n) \approx 2/9 \sigma(\pi^+ p \rightarrow \pi^+ p)$$

$$\frac{\sigma_{\text{Tot}}(\pi^+ p)}{\sigma_{\text{Tot}}(\pi^- p)} = \frac{\sigma(\pi^+ p \rightarrow \pi^+ p)}{\sigma(\pi^- p \rightarrow \pi^- p) + \sigma(\pi^- p \rightarrow \pi^0 n)} = \frac{1}{1/9 + 2/9} = 3$$

$E_{\text{cm}} \approx 1232 \text{ MeV}$

experiment: $\sigma_{\text{Tot}}(\pi^+ p) \sim 195 \text{ mb}$ close to the Δ resonance
 $\sigma_{\text{Tot}}(\pi^- p) \sim 68 \text{ mb}$

□ G-parity

$$C|\pi^\pm\rangle = \alpha|\pi^\mp\rangle \quad (\pi^\pm \text{ not eigenstates of } C)$$

but $|\pi^\pm\rangle \rightarrow |\pi^\mp\rangle$ also by rotation by π around axis I_2 : $e^{i\pi I_2}$
 and (for strong interactions) such isospin rotation is also a symmetry

$$\boxed{\text{G-parity} \quad G = C e^{i\pi I_2}}$$

All mesons (containing quarks u, d) are eigenstates of G

For isospin I : $\boxed{G = C (-1)^I}$
 $\uparrow$ rotation by π around axis z
 eigenvalue of C of neutral component

• Example: pion

$$C|\pi^0\rangle = +|\pi^0\rangle$$

$$I=1$$

$$\boxed{G(\pi^0, \pi^\pm) = -1}$$

$$\boxed{G(N \text{ pions}) = (-1)^N}$$

$$\boxed{\text{Notation} \quad J^P C I G}$$

• $\rho(I=1, C=-1) \Rightarrow G = (-1)(-1)^1 = +1$

$$\rho^0 \rightarrow \pi^+ \pi^- \quad \rho^+ \rightarrow \pi^+ \pi^0 \quad \text{but } \rho \not\rightarrow \pi \pi \pi$$

• $\phi(I=0, C=-1) \Rightarrow G = -1$

$$\phi \rightarrow \pi \pi \pi \quad \phi \not\rightarrow \pi \pi$$

□ Baryons are not eigenstates of C , not even neutral ones:

$$C|p\rangle = |\bar{p}\rangle \quad C|n\rangle = |\bar{n}\rangle$$

(while $C|\pi^\pm\rangle = |\pi^\mp\rangle \quad C|\pi^0\rangle = |\pi^0\rangle$)

However states such as $N\bar{N}$ can be eigenstates of C

with $C(N\bar{N}) = (-1)^{L+S}$ as we have seen

so that

$$\boxed{G(N\bar{N}) = (-1)^{L+S+I}}$$

⌈ Note: for an isospinor ($I=1/2$) we have no direct analogy
 $\pi^+ \pi^-$ has $I=0$ or $I=1$ but $N\bar{N}$ has $I=0$ or $I=1$ in the usual space

Note: for an isospinor ($I=1/2$) we have no direct analogy to the behaviour of a vector in the usual space
What is the effect of a rotation by π around the Z axis?

$$e^{i\pi I_2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = [\cos(\pi/2) \mathbb{1} + i \sin(\pi/2) \sigma_2] \begin{pmatrix} 1 \\ 0 \end{pmatrix} = i \sigma_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\begin{cases} e^{i\pi I_2} |p\rangle = -|n\rangle & \text{so } p\bar{n} \text{ gets back to itself with } G \\ e^{i\pi I_2} |n\rangle = +|p\rangle & \text{but } p\bar{p}, n\bar{n} \text{ do not} \end{cases}$$

however $p\bar{p} \pm n\bar{n}$ still get back to itself

• EX. Recall $G|\pi\rangle = -|\pi\rangle$

Particle $\rho(770)$ is a meson with $J^{PC} I^G = 1^{--} I^+$

– How many charge states of ρ do exist?

Consider the (strong) decay $\rho \rightarrow \pi\pi$

– Write the possible values of L, S, I for the two pions

– Why is $\rho^0 \rightarrow \pi^+\pi^-$ possible while $\rho^0 \rightarrow \pi^0\pi^0$ is not?

Classification of hadrons : quark model

Two (for the moment) quarks : u, d with spin $1/2$ and $I=1/2$

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \quad u = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad d = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \begin{aligned} |u\rangle &= |1/2, 1/2\rangle \\ |d\rangle &= |1/2, -1/2\rangle \end{aligned}$$

Hadrons $\begin{cases} \text{Mesons (} I \text{ integer)} & : q\bar{q} \\ \text{Baryons (} I \text{ half-integer)} & : qqq \\ \text{Antibaryons (} I \text{ half-integer)} & : \bar{q}\bar{q}\bar{q} \end{cases}$

• Let's build baryons (qqq)

(1) Combine two quarks $2 \otimes 2 = 1 \oplus 3$ $\begin{cases} I_+ |u\rangle = 0 & I_+ |d\rangle = |u\rangle \\ I_- |u\rangle = |d\rangle & I_- |d\rangle = 0 \end{cases}$

$|uu\rangle = |1, +1\rangle$

$$\begin{cases} I_- |1, +1\rangle = \sqrt{2} |1, 0\rangle \\ I_- |uu\rangle = |du\rangle + |ud\rangle \end{cases} \quad \text{so} \quad |1, 0\rangle = \frac{1}{\sqrt{2}} (|ud\rangle + |du\rangle)$$

$$\begin{cases} I_- |1, 0\rangle = \sqrt{2} |1, -1\rangle \\ I_- \frac{1}{\sqrt{2}} (|ud\rangle + |du\rangle) = \frac{1}{\sqrt{2}} (|dd\rangle + |dd\rangle) = \sqrt{2} |dd\rangle \end{cases} \quad \text{so} \quad |1, -1\rangle = |dd\rangle$$

and also $|0, 0\rangle = \frac{1}{\sqrt{2}} (|ud\rangle - |du\rangle)$

(2) Now add a third quark $(1 \oplus 3) \otimes 2 = (1 \otimes 2) \oplus (3 \otimes 2) = 2 \oplus 2 \oplus 4$

(2) Now add a third quark $(1 \oplus 3) \otimes 2 = (1 \otimes 2) \oplus (3 \otimes 2) =$

(S) $|uuu\rangle = |3/2, +3/2\rangle$ ($= |uu\rangle |u\rangle$) $= \underbrace{2}_{\text{"MA"}} \oplus \underbrace{2}_{\text{"MS"}} \oplus \underbrace{4}_{\text{"S"}}$

$$J I_{-} |3/2, +3/2\rangle = \sqrt{3} |3/2, +1/2\rangle$$

$$|I_{-} 1 u u u\rangle = |d u u\rangle + |u d u\rangle + |u u d\rangle$$

$$\text{so } |3/2, +1/2\rangle = \frac{1}{\sqrt{3}}(|duu\rangle + |udu\rangle + |uud\rangle)$$

$$\int I_{-} |3/2, +1/2\rangle = 2 |3/2, -1/2\rangle$$

$$\begin{cases} I_{-} |3/2, +1/2\rangle = 2 |3/2, -1/2\rangle \\ I_{-} \frac{1}{\sqrt{3}} (|dun\rangle + |udn\rangle + |lud\rangle) = \frac{1}{\sqrt{3}} (|ddu\rangle + |dud\rangle + |ddn\rangle + |lud\rangle + |lud\rangle + |lud\rangle) \end{cases}$$

$$= \frac{2}{\sqrt{3}} (|ddu\rangle + |dud\rangle + |udd\rangle)$$

$$\text{so } |3/2, -1/2\rangle = \frac{1}{\sqrt{3}}(|ddu\rangle + |dud\rangle + |udd\rangle)$$

$$|3/2, -3/2\rangle = |dddd\rangle$$

All these states are symmetric under exchange of any 2 quarks

MA combine $\frac{|ud\rangle - |du\rangle}{\sqrt{2}}$ with $|u\rangle$ or $|d\rangle$

$$|1/2, +1/2\rangle_{(b)} = \frac{|udd\rangle - |duu\rangle}{\sqrt{2}}, \quad |1/2, -1/2\rangle_{(b)} = \frac{|udd\rangle - |dud\rangle}{\sqrt{2}}$$

which are antisymmetric under exchange of the first two quarks $1 \leftrightarrow 2$ but have no definite symmetry under exchange of $1 \leftrightarrow 3$ or $2 \leftrightarrow 3$

MS Combine $(|dd\rangle, \frac{|ud\rangle + |du\rangle}{\sqrt{2}}, |uu\rangle)$ with $|u\rangle$ or $|d\rangle$

to form states with $I_3 = \pm 1/2$ which are orthogonal to $|3/2, \pm 1/2\rangle$

$$|1/2, +1/2\rangle_{(c)} = \alpha \frac{|ud\rangle + |du\rangle}{\sqrt{2}} |u\rangle + \beta |uu\rangle |d\rangle$$

such that $\langle 3/2, +1/2 | 1/2, +1/2 \rangle = 0 \Rightarrow \beta = -\sqrt{2}\alpha$
and normalize

$$|\frac{1}{2}, \frac{1}{2}\rangle_{(c)} = \frac{1}{\sqrt{6}}(2|uud\rangle - |udu\rangle - |duu\rangle)$$

and $|\frac{1}{2}, -\frac{1}{2}\rangle_{(c)} = \frac{-1}{\sqrt{6}} (2|ddu\rangle - |udd\rangle - |dud\rangle)$

which are symmetric under $1 \leftrightarrow 2$ and have no symmetry for $1 \leftrightarrow 3, 2 \leftrightarrow 3$

□ Now note that quarks are fermions so must be in a totally antisymmetric state under interchange if they are identical

But we are now considering (u, d) as different states of the same

particle q so we write

$$\psi(qqq) = \underbrace{\psi(\vec{r}_1, \vec{r}_2, \vec{r}_3)}_{\text{space}} \underbrace{\chi(s_1, s_2, s_3)}_{\text{spin}} \underbrace{\phi(\alpha_1, \alpha_2, \alpha_3)}_{\text{flavour}} \quad (\alpha = u \text{ or } d)$$

exchanging space coordinates, spins and flavour (u, d)

is equivalent to exchanging two quarks and must give (-1) :

"Generalized Pauli principle"

But consider the particle Δ , with spin $3/2$ and $I=3/2$

- spin $3/2$: there is a state with $S_3 = +3/2$, i.e. $\uparrow\uparrow\uparrow$ symmetric
- isospin $3/2$: there is a state with $I_3 = +3/2$, i.e. uuu symmetric (called Δ^{++})
- being the lowest mass particle with this composition, it has $L_1 = L_2 = L_3 = 0$ symmetric

$\Rightarrow$ state is symmetric under exchange, not possible

$\Rightarrow$ there must be another property of quarks

for which the composition is antisymmetric: color