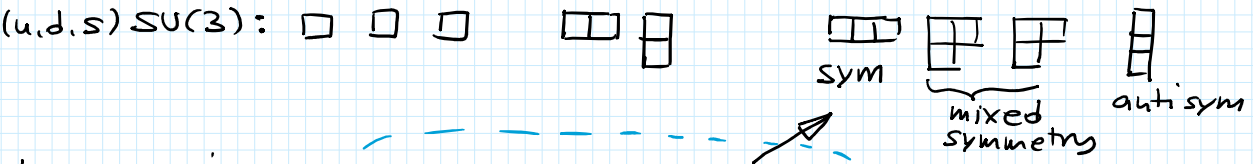
FUNDAM. INT.

Monday, November 9, 2026 10:35 AM

Construction of baryons

3 flavours (u,d,s) SU(3): $3 \otimes 3 \otimes 3 = (6 \oplus \bar{3}) \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$ (999)

← identical particles must be antisymmetric



$\psi_{space}(\vec{r}_1,\vec{r}_2,\vec{r}_3) \cdot \chi_{spin}(\uparrow\downarrow,\uparrow\downarrow,\uparrow\downarrow) \cdot \phi_{flavour}(u,d,s) \cdot$ (colour part antisymmetric)

symmetric

$L_{12} = L_3 = 0$ symmetric (for the lowest mass multiplets)

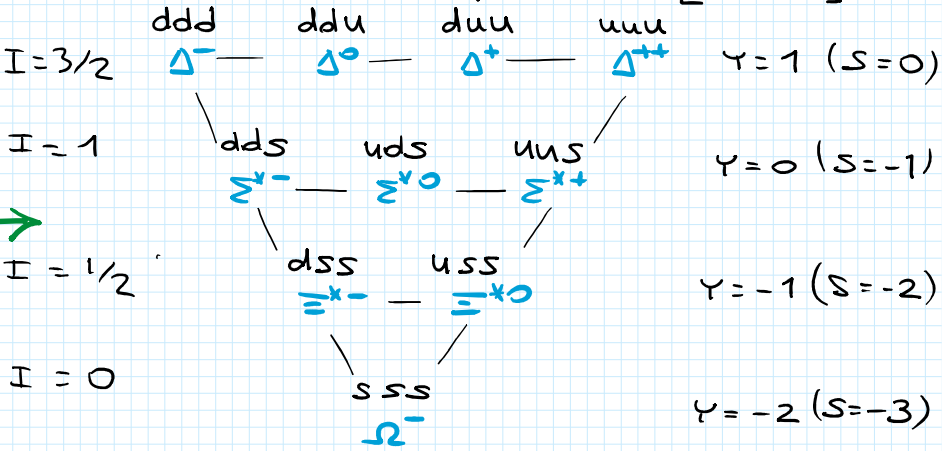
SU(2): $2 \otimes 2 \otimes 2 = (3 \oplus 1) \otimes 2 =$ sym anti sym $2 \oplus 2 \oplus 4$ mixed symm.

This is really $(duu + udu + uud)/\sqrt{3}$ etc.

[$Y = B + S$]

- ϕ: 10 (symmetric)
- χ: $\uparrow\uparrow\uparrow \Rightarrow S_3 = +3/2 \Rightarrow S = 3/2$ (symmetric)

Baryon decuplet with spin 3/2 →



- A fully symmetric wavefunction can also be obtained by combining the mixed-symmetry spin (2) and flavour (8) representations

"$\chi_{(1\leftrightarrow2)(2\leftrightarrow3)(1\leftrightarrow3)}$" "$\phi_{(1\leftrightarrow2)(2\leftrightarrow3)(1\leftrightarrow3)}$"

{sym for 1↔2, antisym for 1↔3} → SU(2) χ_{S-A}

SU(3) ϕ_{S-A} ← {sym for 1↔2, antisym for 1↔3}

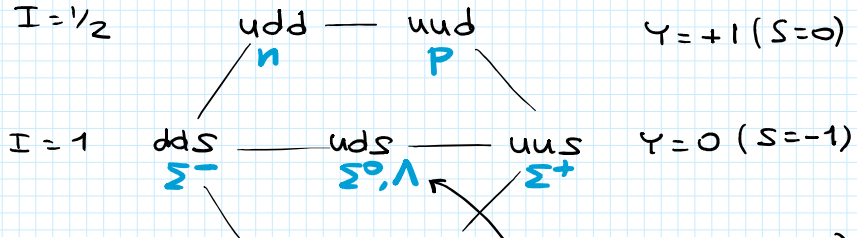
or viceversa

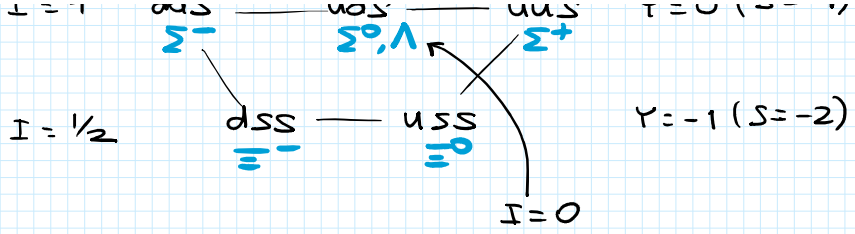
χ_{A-S} ϕ_{A-S}

$$\chi\phi = \frac{1}{\sqrt{2}}(\chi_{S-A}\phi_{S-A} + \chi_{A-S}\phi_{A-S})$$

fully symmetric (also for 2↔3 : show it)

Baryon octet with spin 1/2 →





☐ The Antisymmetric 1 ϕ cannot be used because there is no antisymmetric χ to combine with

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The part which makes the overall wavefunction fully antisymmetric is the colour wavefunction.

Each quarks comes in 3 colors ("r", "g", "b") and all observable particles are colour singlets, totally antisymmetric in colour:

$$qqq: \quad \underbrace{\underbrace{\psi_{space}\chi_{spin}\phi_{flavour}}_{\text{symmetric}}\underbrace{\Psi_{colour}}_{\text{antisymmetric}}}_{\text{antisymmetric}}$$

$$\Psi_{colour} = (rgb - rbg - grb + gbr + brg - bgr)/\sqrt{6}$$

Quantum chromodynamics (QCD): colours are three charges of a gauge field theory describing strong interactions

QED

Symmetry under transformations of the group U(1) complex phases changing from place to place $\chi(x)$ local gauge transformation

$$\psi \rightarrow \psi' = e^{ie\chi(x)}\psi$$

ψ: wavefunction (or field) (single spinor)

coupling of ψ to EM field by

$$\partial_\mu \rightarrow \partial_\mu + ieA_\mu(x)$$

A_μ: photon

Symmetry holds if A_μ transforms as

$$A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu\chi(x)$$

when $\psi \rightarrow \psi'$

QCD

$(a = 1, 2, \ldots 8)$ 3×3 matrices (generators)

strong coupling constant

$$\psi \rightarrow \psi' = e^{igx_a(x)T_a}\psi$$

ψ: wavefunction (or field) $= \begin{pmatrix} r \\ g \\ b \end{pmatrix}$ (three spinors)

coupling of ψ to colour field by

$$\partial_\mu \rightarrow \partial_\mu + igT_a G_{a\mu} \quad (a = 1, \ldots 8)$$

$G_{a\mu}$: 8 gluons

$$G_{\mu a} \rightarrow G'_{\mu a} = G_{\mu a} - \partial_\mu\chi_a(x) - gf_{abc}\chi_b(x)G_{c\mu}$$

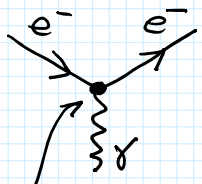
$(a, b, c = 1 \ldots 8)$

This generalization of EM to a symmetry under a non-abelian group such as SU(3) is called

This generalization of QM to a symmetry under a non-abelian group such as SU(3) is called

Yang-Mills theory

Vertex:



Feynman rule

$-ie\gamma^\mu$

q_i q_j $i,j = r,g,b$ (quark colour)

g_a (gluon) $(a = 1, \dots 8)$

Quarks colour changes the gluons carry colour

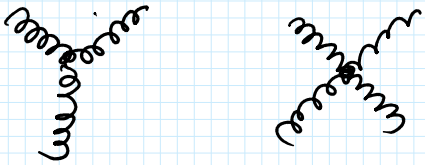
Feynman rule

$-ig\gamma^\mu (T_a)_{ij}$

γ is neutral, it does not interact directly with other photons

gluons are coloured, they interact directly with other gluons

Two other vertexes exist:



Note: the colour group $SU(3)_{colour}$ describing the exact colour symmetry has nothing to do with the $SU(3)_{flavour}$ describing the approximate flavour symmetry when only 3 quark flavours (u, d, s) are considered

They just share the same mathematics (like spin & isospin)

Colour confinement principle:

all physical hadrons are colour singlets

This is possible for: $SU(3)_{colour}$

- qqq $\quad 3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$ ← singlet

  baryons

  (and for $\bar{q}\bar{q}\bar{q}$, antibaryons)

- $q\bar{q}$ $\quad 3 \otimes \bar{3} = 8 \oplus 1$ ← singlet

  mesons

but for example not for qq : $3 \otimes 3 = 6 \oplus \bar{3}$ no singlet

Dynamical "explanation":

the strong colour interaction grows with increasing distance

when a hadron breaks into quarks, as they move apart their interaction energy increases up to the point when it can create a $q\bar{q}$ pair and quarks recombine in colourless

(singlet) hadrons

