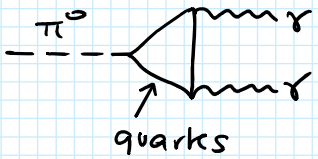
FUNDAM. INT.

Wednesday, November 18, 2026 10:45 AM

Further evidence for color

$\pi^0 \to \gamma\gamma$



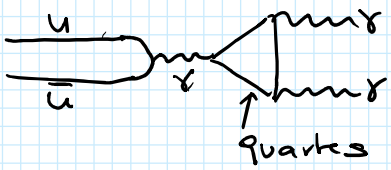
γ cannot couple directly to a neutral particle

If 3 possible colours : amplitude $\times 3$

$|\mathcal{M}|^2 \times 9$ = experiment

In terms of quarks :

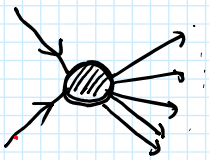
$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \to$$

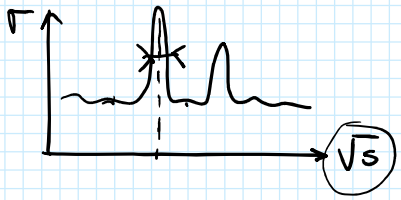


or $d\bar{d}$

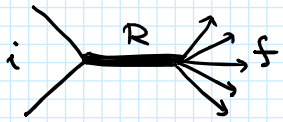
Resonances

Process $i \to f$





Contribution by resonance R (unstable particle)



$$\psi(t) = \psi(0)\, e^{-iMt}\, e^{-(\Gamma/2)t} = \psi(0)\, e^{-it(M - i\Gamma/2)}$$

Time evolution of unstable particle

$$|\psi(t)|^2 = |\psi(0)|^2\, e^{-\Gamma t} = |\psi(0)|^2\, e^{-t/\tau} \qquad (\Gamma = \hbar/\tau)$$

Initial state $|\psi_i(t)\rangle = |i\rangle\, e^{-iEt}$

Final state $|\psi_f(t')\rangle = |f\rangle\, e^{-iEt'}$

$(E = E')$ $(t' > t)$ (t': decay, t: production)

$$|\psi_R(t)\rangle = |R\rangle\, e^{-i(M - i\Gamma/2)t}$$

$$\mathcal{M} = \langle \psi_f(t') | \psi_i(t) \rangle = \langle f | i \rangle\, e^{-iE(t - t')}$$

Through resonance:

$$\mathcal{M}_R = \int_0^\infty d(t'-t)\, \langle \psi_f(t') | \psi_R(t') \rangle \langle \psi_R(t') | \psi_R(t) \rangle \langle \psi_R(t) | \psi_i(t) \rangle$$

(decay, evolution of R, prod.)

$$= \langle f|R\rangle \langle R|i\rangle \int_0^\infty d(t'-t)\, e^{iE(t'-t)}\, e^{-i(t'-t)(M - i\Gamma/2)} =$$

$$= \langle f|R\rangle \langle R|i\rangle \frac{-i \langle f|R\rangle \langle R|i\rangle}{E - M + i\Gamma/2}$$

$$\sigma \propto |\mathcal{M}_R|^2 = \frac{|\langle f|R\rangle|^2\, |\langle R|i\rangle|^2}{(E-M)^2 + \Gamma^2/4}$$

Complete formula for resonance of spin J in reaction with unpolarized particles $\{1,2\} = i$ with spins S_1, S_2 :

$\Gamma_i \equiv \Gamma(R \to i) = \Gamma \cdot BR(R \to i)$
$\Gamma_f \equiv \Gamma(R \to f) = \Gamma \cdot BR(R \to f)$

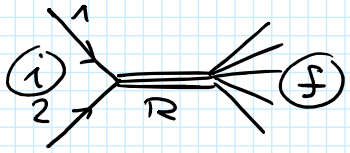
$$\sigma(i \to R \to f) = \frac{4\pi}{k^2} \frac{(2J+1)}{(2s_1+1)(2s_2+1)} \frac{\Gamma_i \Gamma_f / 4}{(E-M)^2 + \Gamma^2/4}$$

Breit-Wigner

k: momentum of particle 1 or 2 in CM

☐ This describes resonance in formation

evidence as peak in $\sigma_{if}(\sqrt{s})$
or $\sigma_{if}(E_1^{LAB})$

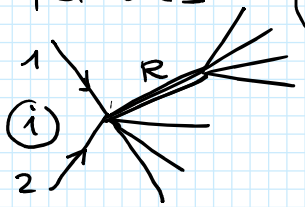


Formation is limited to resonances of mass $M_R \geq m_1 + m_2$

e.g. strange resonances produced by strong interactions need $S \neq 0$ initial state, e.g. $K^- p$ but this only allows to form resonances with $M_R > 1431$ MeV

and not e.g. $\Sigma(1385)$

☐ Production: resonance is produced together with other particles and one studies the invariant mass distribution of a subset of final state particles



$$M_{INV} = \sqrt{(\Sigma_i E_i)^2 - (\Sigma_i \vec{p}_i)^2} \quad (i = 1 \ldots N)$$