FUND. INTER.

Wednesday, November 25, 2026 10:47 AM

Fermi interaction:

$$(\bar{\psi}_b \gamma^\mu \psi_a)(\bar{\psi}_c \gamma_\mu \psi_d)$$

☐ more general:

$$[\bar{\psi}(\ )\psi][\bar{\psi}(\ )\psi]$$

		Non-relativistic limit appropriate to β decay	($\vec{p}_N \ll E_N$) surviving terms
S:	$\bar{\psi}\psi$	→	$\psi^+\psi$
V:	$\bar{\psi}\gamma^\mu\psi$	→	$\psi^+\psi$
T:	$\bar{\psi}[\gamma^\mu,\gamma^\nu]\psi$	→	$\psi^+\sigma\psi$ (Pauli matrix)
A:	$\bar{\psi}\gamma^\mu\gamma_5\psi$	→	$\psi^+\sigma\psi$
P:	$\bar{\psi}\gamma_5\psi$	→	0

two component spinor

{ S, V describe β decays in which $\Delta J = 0$

A, T " " " $\Delta J = 1$ (σ present)

both exist → need at least one of (S, V) and one of (A, T)

study of energy spectrum → only one of each pair

study of lifetimes of decays with $\Delta J = 0$ or $\Delta J = 1$

→ only V, A with same magnitude and opposite sign

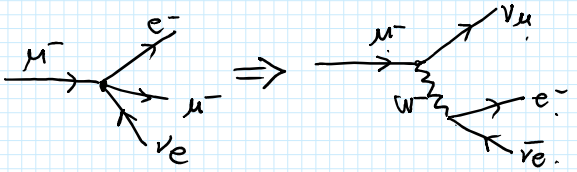
$$J^\mu \propto \bar{\psi}(\gamma^\mu - \gamma^\mu\gamma_5)\psi = \bar{\psi}\gamma^\mu(1-\gamma_5)\psi \qquad \text{V-A theory}$$

Modified Fermi Hamiltonian:

$$H = \frac{G_F}{\sqrt{2}}(\bar{\psi}_p \gamma^\mu(1-\gamma_5)\psi_n)(\bar{\psi}_e \gamma_\mu(1-\gamma_5)\psi_\nu)$$

to keep same value of G_F as before

☐ Point-like interaction → mediator particle



$W^\pm$ $J = 1$

$m_W \neq 0$ (short-range)

$$H = \frac{g}{\sqrt{2}}(\bar{\psi}_{\nu_\mu}\gamma^\mu\tfrac{1}{2}(1-\gamma_5)\psi_\mu)\,\frac{g_{\mu\nu} - q_\mu q_\nu/m_W^2}{q^2 - m_W^2}\,\frac{g}{\sqrt{2}}(\bar{\psi}_e\gamma^\nu\tfrac{1}{2}(1-\gamma_5)\psi_{\nu_e})$$

For $|q^\mu| \ll m_W$: $\to -g_{\mu\nu}/m_W^2$

so that

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

knowing now that

$m_W = 80.4$ GeV

$$\alpha_W = \frac{g^2}{4\pi\hbar c} \simeq \frac{1}{30} > \alpha = \frac{1}{137}$$

The weakness of weak int. at low energies is due to large m_W

☐ The (charged) weak current changes flavour

$$J_W^\mu = \frac{g}{\sqrt{2}}\bar{\psi}_b\gamma^\mu\tfrac{1}{2}(1-\gamma_5)\psi_a \qquad (a \neq b)$$

in particular $d \longleftrightarrow u$ (*) $\quad \mu^- \longleftrightarrow \nu_\mu$

n decay: udd $\quad e^- \longleftrightarrow \nu_e$

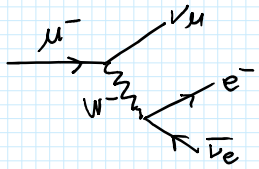
in particular $d \longleftrightarrow u^{(*)}$ $\mu^- \longleftrightarrow \nu_\mu$

n decay : udd $\hookrightarrow u$ $e^- \longleftrightarrow \nu_e$

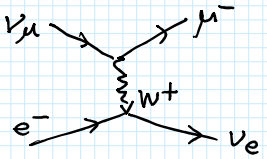
μ decay : $\mu \hookrightarrow \nu_\mu$ $\tau^- \longleftrightarrow \nu_\tau$

(*) more news on this later

- Leptonic processes:

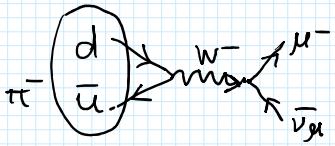


μ decay



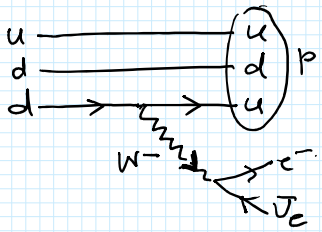
ν scattering

- Leptonic decays of hadrons

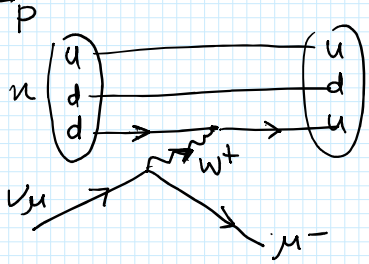
π decay π^- 

(also $e^- \bar{\nu}_e$)

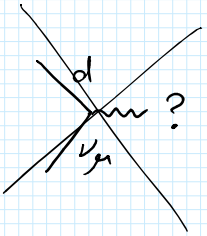
- Semi-leptonic decays

β decay $n \to p e^- \bar{\nu}_e$ 

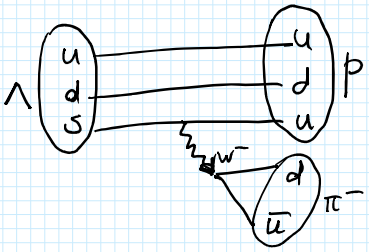
is really: $d \to u e^- \bar{\nu}_e$

ν scattering on hadrons $\nu_\mu n \to \mu^- p$ 

$\nu_\mu d \to u \mu^-$



- Non-leptonic decays

$\Lambda \to p \pi^-$ 

☐ μ decay

$\Gamma_\mu = \tau_\mu^{-1} \propto G_F^2 (\ldots)$

Dimensional considerations:

$$[\Gamma] = [E] = [G_F]^2 [(\ldots)]$$

$[E]^{-4}$ $[E]^5$

$\mu^- \to e^- \nu_\mu \bar{\nu}_e$

$m_\mu = 106$ MeV

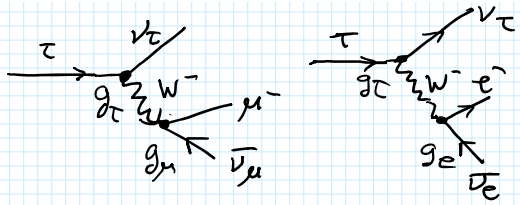
$m_e = 0.5$ MeV

For $\mu^- \to e^- \nu_\mu \bar{\nu}_e$ the only quantities with dimensions of $[E]$ are the masses $m_\mu \gg m_e \to \Gamma \propto^2 G_F^2 m_\mu^5$

$$\Gamma = \frac{1}{192\pi^3} G_F^2 m_\mu^5 (1+\varepsilon)$$ ← m_e...

•

Compare $\begin{cases} \tau^- \to \mu^- \nu_\tau \bar{\nu}_\mu \\ \tau^- \to e^- \nu_\tau \nu_e \end{cases}$



$$\begin{cases} \Gamma(\tau^- \to \mu^- \nu_\tau \bar{\nu}_\mu) \propto \dfrac{g_\tau^2 g_\mu^2}{(M_W^2)^2} m_\tau^5 (\dots) \to p_\mu^2 \\ \Gamma(\tau^- \to e^- \nu_\tau \bar{\nu}_e) \propto \dfrac{g_\tau^2 g_e^2}{(M_W^2)^2} m_\tau^5 (\dots) \to p_e^2 \end{cases}$$

$$\frac{\Gamma(\tau^- \to \mu^- \nu_\tau \bar{\nu}_\mu)}{\Gamma(\tau^- \to e^- \nu_\tau \bar{\nu}_e)} = \frac{g_\mu^2 p_\mu^2}{g_e^2 p_e^2} = \frac{BR(\tau^- \to \mu^- \nu_\tau \bar{\nu}_\mu)}{BR(\tau^- \to e^- \nu_\tau \bar{\nu}_e)} \to \frac{g_\mu}{g_e} = 1.001 \pm 0.002$$

And g_τ?

$BR(\mu^- \to e^- \nu_\mu \bar{\nu}_e) = 1$

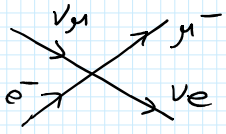
$$\frac{\Gamma(\mu^- \to e^- \nu_\mu \bar{\nu}_e)}{\Gamma(\tau^- \to e^- \nu_\tau \bar{\nu}_e)} = \frac{\Gamma_\mu BR(\dots)}{\Gamma_\tau BR(\dots)} = \frac{\tau_\tau}{\tau_\mu} \frac{BR(\dots)}{BR(\dots)}$$

$$= \frac{g_\mu^2}{g_\tau^2} \frac{m_\mu^5}{m_\tau^5} \frac{(\dots) \to p_e^{(\mu)}}{(\dots) \to p_e^{(\tau)}} \to \frac{g_\mu}{g_\tau} \simeq 1$$

lepton universality of weak interactions

□ Neutrino scattering

$\nu_\mu e^- \to \mu^- \nu_e$



• Using Fermi theory:

Dimensional argument: $\sigma \propto G_F^2 (\dots)$

$[\sigma] = [L]^2 = \frac{1}{(E)^2}$ $[G_F] = \frac{1}{(E)^2}$ $[(\dots)] = [E]^2$

At high energies $\sqrt{s} \gg m_\mu \gg m_e$

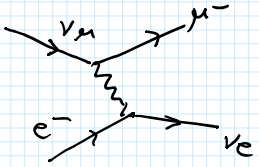
it must be $\sigma \propto G_F^2 s$ $\left[\sigma = \frac{G_F^2}{\pi} s \right.$

where $s = (p_\nu + p_e)^2 = \cancel{m_\nu^2} + m_e^2 + 2 p_\nu p_e$

(in LAB): $= m_e^2 + 2 E_\nu m_e \simeq 2 E_\nu m_e$

so that $\sigma \propto E_\nu$ (in the lab) → cannot be correct for $E_\nu \to \infty$

• Using theory with $W^\pm$ when $E_\nu \gtrsim m_W$ → no divergence for $E_\nu \to \infty$



□ chiral structure of weak interactions

The weak currents are $j^\mu = \bar{\psi} \gamma^\mu \frac{1}{2}(1 - \gamma_5) \psi$

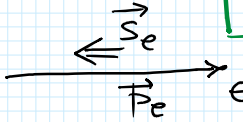
Chiral structure of weak interactions

The weak currents are $j^\mu = \bar{\psi}\gamma^\mu \frac{1}{2}(1-\gamma_5)\psi$

Recall that $\psi = \psi_R + \psi_L$ $\quad \begin{cases} \psi_L = \frac{1}{2}(1-\gamma_5)\psi & \text{(chirality } -1\text{)} \\ \psi_R = \frac{1}{2}(1+\gamma_5)\psi & \text{(chirality } +1\text{)} \end{cases}$

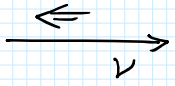
$j^\mu = \bar{\psi}_L \gamma^\mu \psi_L$ (show)

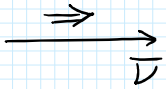
Remember: for ultra-relativistic particles chirality $\simeq$ helicity
" " antiparticles chirality $\simeq$ $-$helicity

$\psi_L =$  for high-energy electrons

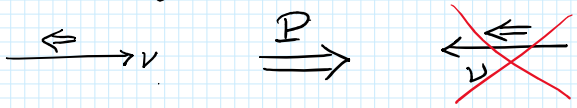
ψ_R does not interact with charged weak currents

at high energies only (helicity $= -1$) electrons are involved in weak interactions

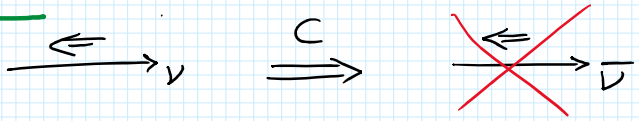
ν are always ultra-relativistic ($m_\nu \simeq 0$) and only interact weakly so only  exist



Maximal parity violation:

- coefficients of γ^μ and $\gamma^\mu\gamma_5$ have same magnitude (parity-violating interference term maximal)
-  which does not exist

C-violation:

 which does not exist
C maximally violated

However  whis exists

CP conserved

1964: weak interactions also violate CP, but by a small amount totally insufficient to explain $\frac{n_B - n_{\bar{B}}}{n_B + n_{\bar{B}}} \simeq 10^{-9} \neq 0$ in universe

$\Rightarrow$ OPEN PROBLEM

π decay

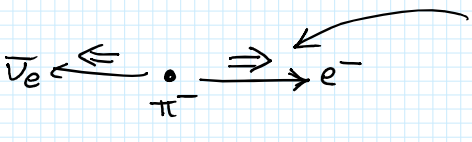
$\begin{cases} \pi^- \to e^- \bar{\nu}_e \\ \pi^- \to \mu^- \bar{\nu}_\mu \end{cases}$ but despite universality $\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \simeq 1.2 \cdot 10^{-4}$

the decay mode with smaller phase space volume is strongly favoured

Reason: chiral structure

the decay mode with smaller phase space volume is strongly favoured

Reason: chiral structure



To have $J_z = 0$ (pion has spin 0)
e^- must have helicity $+1$
but only the ψ_L part is involved

$$u(h=+1) = \frac{1}{2}\left(1 + \frac{|\vec{p}|}{E+m}\right) u_R + \frac{1}{2}\left(1 - \frac{|\vec{p}|}{E+m}\right) u_L$$

if $|\vec{p}| \sim E \gg m \quad u(h=+1) = u_R$

$\mathcal{H}_l \propto \frac{1}{2}\left(1 - \frac{|\vec{p}|}{E+m}\right) \longrightarrow 0$ for relativistic particle

The electron is much more relativistic than the μ in π decay

In CM of π: $E_l = \frac{m_\pi^2 + m_l^2}{2m_\pi} \quad |\vec{p}_l| = \frac{m_\pi^2 - m_l^2}{2m_\pi} = \frac{m_\pi}{2}\left(1 - \frac{m_l^2}{m_\pi^2}\right)$

$$\frac{|\vec{p}_l|}{E_l + m} = \frac{m_\pi - m_l}{m_\pi + m_l} \qquad \mathcal{H}_l \propto \frac{m_l^2}{(m_\pi + m_l)^2}$$

Complete computation $|\mathcal{H}_l|^2 \propto G_F^2 m_l^2 \underbrace{(m_\pi^2 - m_l^2)}_{P_l}$ phase space

$$\frac{\Gamma(\pi \to e\nu)}{\Gamma(\pi \to \mu\nu)} = \frac{m_e^2 (m_\pi^2 - m_e^2)^2}{m_\mu^2 (m_\pi^2 - m_\mu^2)^2} = 1{,}2 \cdot 10^{-4}$$